Optimal Monetary Policy in a Model with Customer Markets

Özge Akinci and Ryan Chahrour *

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Abstract

We investigate the implications of habits in consumption for optimal monetary policy. Using a baseline model which nests four popular specifications of habit ([relative, additive] x [deep, superficial]), we derive three main results: 1) in a flexible-price habit model, the optimal linear labor subsidy/tax is time-varying; 2) in a sticky-price habit model, given any constant labor subsidy, monetary policy cannot achieve a pareto optimal allocation; and 3) the implications for inflation under optimal policy depend heavily on the type of habit. In particular, the additive-deep specification implies high inflation volatility in a model driven by technology shocks. We argue that the key model feature driving this result is time-variation in the short-run price elasticity of demand.

^{*}This research supported by the Program for Economic Research at Columbia University. The authors thank Stephanie Schmitt-Grohé, Martín Uribe and seminar participants at Columbia University for many helpful conversations. Columbia University, Department of Economics, International Affairs Building, 420 W. 118th St., New York, NY 10027. Özge Akinci email: oa2140@columbia.edu; Ryan Chahrour email: rc2374@columbia.edu

1 Introduction

In recent years, New Keynesian (NK) models with sticky prices have become a workhorse of monetary policy analysis (Rotemberg and Woodford (1997), Woodford (2003), Schmitt-Grohé and Uribe (2007)). This work has yielded at least one consistent result: optimal monetary policy implies perfect or almost perfect price stability. Recent work adds labor and credit market frictions to the NK models, without significantly altering the price-stability result (Ravenna and Walsh (2009); Curdia and Woodford (2008)). Meanwhile, surprisingly little attention is given to the normative consequences of goods market frictions, despite evidence that they capture important features of the macroeconomy (Ravn et al. (2006, 2008)). In this paper, we examine the implications of one such friction, habits in consumption, for optimal monetary policy. We find that implications for monetary policy depend crucially on how habits are modelled. In particular, optimal inflation volatility is high when consumer utility depends on the quasidifference of current consumption and a good-specific habit stock (additive-deep habits). In contrast, when habits depend only on past aggregate consumption (superficial habit), or utility depends on the quasi-ratio of current consumption to a habit stock (relative habits), optimal inflation volatility remains quite low.

Time-varying markups are the key feature driving our results. As discussed in Rotemberg and Woodford (1999), mechanisms which generate variable markups typically fall into one of two categories: either, 1) firms' desired markups vary over time or 2) frictions prevent firms from achieving a (constant) desired markup. Models of the second type (e.g. models with nominal rigidities) have been widely studied in the optimal policy literature, but relatively little work examines optimal policy when desired markups vary endogenously. Because deep habit models generate endogenous desired markup variation, they may have very different implications for optimal monetary policy. Authors studying simple NK models have found that, with a constant employment subsidy designed to offset the markups of monopolistically competitive firms, perfect price stability is the optimal monetary policy. In the case with deep habits, however, the subsidy required to offset markups is time-varying, even under optimal policy. Furthermore, without a subsidy (or with any constant subsidy), optimal monetary policy can deviate substantially from price stability; a result that contrasts with the finding of essentially stable prices in models without habits.

Because our model nests a variety of habit specifications, we can isolate the mechanism that leads to substantial optimal inflation volatility in the additive-deep case. Leith et al. (2008) argue that increased inflation volatility is explained by the choice to model habits as external: the monetary authority should accept inflation in order to offset the externality created by consumers who ignore their impact on habit stock. We eliminate this explanation, however, by specifying a time-varying subsidy which offsets only markups. With this subsidy in place, a zero inflation policy yields the same allocation as the flexible price *competitive* (no markup) equilibrium, which is not pareto efficient if habits are external. Depsite the inefficiency in this case, optimal policy yields zero inflation and welfare exactly equal to welfare under the flexible price competitive equilibrium. If the externality explanation were true, welfare under optimal policy should go at least part way to bridging the gap between the social planner (pareto efficient) allocation and the flexible price equilibrium. More generally, with no subsidy, optimal inflation volatility is quite low in all but one of the habit specifications we consider, suggesting that the habit externality alone cannot generate substantial inflation volatility.

Time-varying markup is the second candidate explanation of relatively high optimal inflation volatility in our model. With deep habits, desired markups may vary for two reasons. First, regardless of whether habits are relative or additive, firms face a dynamic (customer markets) pricing problem. Because firms care about the size of their customer base, and the value of this base varies over time, their willingness to sacrifice profits today for future customers is also time-varying. Second, uniquely in the case of additivedeep habits, the short-run elasticity of demand varies over time. We show that optimal inflation volatility is high only in the case of additive-deep habits, suggesting that it is the second mechanism (time varying elasticity of demand) which generates substantial inflation volatility.

Our model builds on the most simple version of the NK model, by adding a goods-market friction via habits. We study optimal monetary policy with both aggregate (superficial) and good-specific (deep) habits, following the ramsey approach of Schmitt-Grohé and Uribe (2004). Their approach, which avoids the need to approximate the welfare objective, allows us to employ a variety of linear labor subsidies to isolate how each inefficiency affects the design of optimal monetary policy.

The literature introducing customer markets via good-specific habits (deep habits) into the DSGE environment is motivated by two stylized facts of the macroeconomy: countercyclical markups, and procyclical real wages. Ravn et al. (2006) propose a general equilibrium model of endogenously varying markups by introducing habit formation over specific goods in an economy with imperfectly competitive product markets. In their model, the single-period utility function depends on the quasi-difference of current consumption and the stock of habit for each good (additive deep habits). For a given level of habit in this model, the price elasticity of an individual good is increasing in the quantity demanded. This feature distinguishes deep habits from habits over the composite consumption good (superficial habits). In this environment, a firm expecting high demand in the future has an incentive to build market share today by reducing current markups. Through these mechanisms, Ravn et al. (2006) are able to match some key stylized facts. Recent work incorporates nominal frictions into this environment. Ravn et al. (2008) find that the countercyclical markup movements induced by deep habits are helpful for accounting for the dynamic effects of monetary policy shocks, even at low to moderate levels of nominal rigidities.

2 The Model

We present a simple model, without capital formation, in which habits are good-specific (deep) and additive. We briefly introduce relative deep habits at the end of this section to analyze and compare the mechanisms through which each model generates time varying markups.

2.1 Consumers

Our economy is populated by a continuum of identical consumers of measure one indexed by $j\epsilon[0,1]$ with utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^j, h_t^j) \tag{1}$$

where

$$x_t^j = \left[\int (C_{i,t}^j - \theta S_{i,t-1})^{1-\frac{1}{\eta}} di\right]^{\frac{1}{1-\frac{1}{\eta}}}$$
(2)

$$S_{i,t} = \rho S_{i,t-1} + (1-\rho)C_{i,t}$$
(3)

and η denotes intertemporal elasticity of substitution between good varieties, $i\epsilon[0,1]$, of habit adjusted consumption. The consumer faces a sequential budget constraint given by

$$\int_{0}^{1} P_{i,t} C_{i,t}^{j} di + B_{t}^{j} + T_{t}^{j} = R_{t-1} B_{t-1}^{j} + (1 - \tau_{t}) W_{t} h_{t}^{j} + \Phi_{t}^{j}$$

$$\tag{4}$$

In each period $t \ge 0$, consumers have access to a risk free nominal bond B_t that pays the gross nominal interest rate R_t in period t + 1. Consumers take the nominal wage W_t as given. In addition, consumers are subject to a borrowing constraint that prevents them from engaging in Ponzi financing.

For any given level of x_t^j , purchases of each variety, *i*, in period *t*, must solve the dual problem of minimizing total expenditures. From the cost minimization problem we derive consumer *j*'s demand for

good i as

$$C_{i,t}^{j} = \left(\frac{P_{i,t}}{P_{t}}\right)^{-\eta} x_{t}^{j} + \theta S_{i,t-1}$$

$$\tag{5}$$

where $P_t \equiv \left(\int P_{i,t}^{1-\eta} di\right)^{\frac{1}{1-\eta}}$ defines the nominal price index. At the optimum, $P_t x_t^j = \int P_{i,t} (C_{i,t}^j - \theta S_{i,t-1}) di$ which can be written as

$$\int P_{i,t}C_{i,t}^{j}di = P_{t}x_{t}^{j} + \theta \int P_{i,t}S_{i,t-1}di$$
(6)

Consumers solve

$$\max_{\{x_t^j, h_t^j, B_t^j\}} E_0 \sum_{t=0}^{\infty} \beta U(x_t^j, h_t^j)$$
(7)

subject to their budget constraint, restated from equation 4

$$P_t x_t^j + B_t^j + T_t + \chi_t = (1 - \tau_t) W_t h_t^j + R_{t-1} B_{t-1}^j + \Phi_t^j$$
(8)

where $\chi_t = \theta \int P_{it} S_{it-1} di$, T_t is a lump sum tax, and τ_t is a time-varying employment subsidy. Note that, in general, nothing prevents T_t or τ_t from being less than zero, in which case they would represent respectively a subsidy and a tax.

The first order conditions of the household's problem are:

$$\frac{(1-\tau_t)W_t}{P_t} = -\frac{U_h(x_t^j, h_t^j)}{U_x(x_t^j, h_t^j)}$$
(9)

$$1 = E_t \left[\beta \frac{U_x(x_{t+1}^j, h_{t+1}^j)}{U_x(x_t^j, h_t^j)} \frac{P_t}{P_{t+1}} R_t \right]$$
(10)

2.2 Firms

Goods are produced by monopolistic firms. Each good is produced via a linear production function, $y_{i,t} = A_t h_{i,t}$, where A_t is an exogenous processes for productivity, $log(A_{t+1}/\bar{A}) = \rho_a log(A_t/\bar{A}) + \varepsilon_{t+1}^a$. We assume prices are sticky à la Rotemberg (1982).

Firms solve

$$\max_{P_{i,t}, C_{i,t}, S_{i,t}, h_{i,t}} E_0 \sum_{t=0}^{\infty} q_t \left[P_{i,t} C_{i,t} - W_t h_{i,t} \right]$$

subject to

$$C_{i,t} = A_t h_{i,t} - \frac{\varphi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - \pi_t^* \right)^2$$
(11)

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} x_t + \theta S_{i,t-1}$$
(12)

$$S_{i,t} = \rho S_{i,t-1} + (1-\rho)C_{i,t}$$
(13)

given exogenous processes q_t, A_t, P_t and $x_t = \int x_t^j dj$. The variable $q_t = \beta^t U_x(x_t, h_t)/P_t$, derived from households problem after imposing homogeneity across consumers, is a pricing kernel determining the period zero utility value of one unit of composite good delivered in a particular state at period t. We let the (real) multipliers on the constraints be called λ_t, μ_t, ξ_t , respectively.

The first order conditions of the firms' problem are given by

$$\lambda_t = \frac{W_t}{P_t A_t} \tag{14}$$

$$\mu_t = \frac{P_{i,t}}{P_t} - \lambda_t + (1-\rho)\xi_t \tag{15}$$

$$C_{i,t} - \eta \mu_t \frac{P_t}{P_{i,t}} (C_{i,t} - \theta S_{i,t-1}) = \varphi \lambda_t (\pi_{i,t} - \pi_t^*) \frac{P_t}{P_{i,t-1}} - \varphi \beta E_t \left[\lambda_{t+1} \frac{q_{t+1}}{q_t} (\pi_{i,t+1} - \pi_{t+1}^*) \pi_{i,t+1} \frac{P_{t+1}}{P_{i,t}} \right]$$

$$\xi_t = \beta E_t \left[\frac{q_{t+1}}{q_t} \pi_{t+1} (\theta \mu_{t+1} + \rho \xi_{t+1}) \right]$$
(17)

2.3 Symmetric Equilibrium

We focus on a symmetric equilibrium where all firms charge the same price and consumers consume identical baskets. The symmetric equilibrium (excluding policy rules) is summarized by the following set of equations:

$$x_t = C_t - \theta S_{t-1} \tag{18}$$

$$\omega_t (1 - \tau_t) = -\frac{U_h(x_t, h_t)}{U_x(x_t, h_t)}$$
(19)

$$1 = E_t [\beta \Omega_{t+1} / \pi_{t+1}] R_t \tag{20}$$

$$C_t - \eta \mu_t (C_t - \theta S_{t-1}) = \varphi \lambda_t (\pi_t - \pi_t^*) \pi_t - \varphi \beta E \Omega_{t+1} \lambda_{t+1} (\pi_{t+1} - \pi_{t+1}^*) \pi_{t+1}$$
(21)

$$\mu_t = (1-\rho)\xi_t + 1 - \lambda_t \tag{22}$$

$$\xi_t = \beta E_t \Omega_{t+1} \left(\theta \mu_{t+1} + \rho \left(\xi_{t+1} \right) \right)$$
(23)

$$\lambda_t = \omega_t / A_t \tag{24}$$

$$C_{t} = A_{t}h_{t} - \frac{\varphi}{2} (\pi_{t} - \pi_{t}^{*})^{2}$$
(25)

$$y_t = A_t h_t \tag{26}$$

$$S_t = \rho S_{t-1} + (1-\rho)C_t \tag{27}$$

where $\omega_t \equiv \frac{W_t}{P_t}$ and

$$\Omega_t \equiv \frac{U_x(x_t, h_t)}{U_x(x_{t-1}, h_{t-1})} = \left(\frac{x_t}{x_{t-1}}\right)^{-\sigma}$$
(28)

We assume the exogenous technology process $log(A_{t+1}/\bar{A}) = \rho_a log(A_t/\bar{A}) + \epsilon_{t+1}^A$. In appendix 6.1, we show how to solve analytically for the model's steady-state.

2.4 Fiscal Policy

For isolating the effects of the various inefficiencies on optimal policy (and comparability to earlier studies), we define a variety of subsidy/tax rules that the government may follow. In all the cases we consider, labor is subsidized (taxed) at a linear rate, and subsidies (revenues) are financed by lump-sum taxes (rebates.) The government therefore faces a simple budget constraint: $\tau_t W_t h_t = T_t$.

First, we define the *optimal subsidy*, τ^{opt} , as the subsidy which achieves the pareto optimal allocation for the economy with flexible prices. Second, we define the *markup subsidy* as the subsidy required to achieve the economy's flexible-price perfectly competitive equilibrium. The markup subsidy is implemented by selecting τ_t^{mkp} such that $(1 - \tau_t^{mkp}) = markup_t$. In this case, the only role of fiscal policy is to offset the low-level of output caused by monopolistically competitive firms. Because our model incorporates a habit externality, the flexible price allocation is not pareto optimal, in contrast to other models used in this literature. Furthermore, because our approach to solving the ramsey problem does not require a second-order welfare approximation, we can log-linearize the model, and compare optimal policy, around this inefficient steady-state. Finally, we define the *constant subsidy* rule as that which sets $\tau_t = c$. Because the deep-habits model incorporates time-variation in desired markups, no constant subsidy will be able to offset the monopolistic distortion, even under perfect price stability. This time varying "markup inefficiency" creates the potential for non-zero inflation volatility under optimal policy.

2.5 Other Forms of Habit

We consider three other forms of habit: relative-deep, additive-superficial and relative-superficial. In all cases, the stock of habit, S_t , is assumed to evolve according to the law of motion $S_t = \rho S_{t-1} + (1 - \rho)C_t$.

Relative Deep Habit

In this specification, for each good i, households derive utility from the quasi-ratio of current consumption to a measure of lagged aggregate consumption:

$$x_t^j = \left[\int \left(\frac{C_{i,t}^j}{S_{i,t-1}^{\theta}} \right)^{1-\frac{1}{\eta}} di \right]^{\frac{1}{1-\frac{1}{\eta}}}$$
(29)

The optimal level of consumption, derived from cost minimization problem, is then given by the following expression:

$$C_{i,t} = \left(\frac{\tilde{p}_{i,t}}{\tilde{p}_t}\right)^{-\eta} \left(S_{i,t-1}^{\theta}\right)^{1-\eta} x_t \tag{30}$$

where $\tilde{p}_t = \left[\int \left(\tilde{p}_{i,t} S_{i,t-1}^{\theta} \right)^{1-\eta} di \right]^{\frac{1}{1-\eta}}$ is the relative price of x_t^j in terms of the composite good, $\tilde{p}_{i,t} = \frac{P_{i,t}}{P_t}$, and $P_t = \left[\int P_{i,t}^{(1-\eta)} di \right]^{1/(1-\eta)}$. We call the ratio $\frac{\tilde{p}_{i,t}}{\tilde{p}_t}$ the *externality adjusted price* of the variety *i*. Similar to additive-deep habit specification, the overall demand for good *i* is dynamic and, hence, the firm's pricing decision is also dynamic. The equilibrium conditions determining the markup with relative-deep habits, however, are quite different than with additive-deep habit. We study the entire system, including markup dynamics, in detail in Appendix 6.4.

Additive Superficial Habit

Habits are superficial when they are formed at the level of the aggregate consumption good. Specifically, $x_t^j = C_t^j - \theta S_{t-1}$. The overall demand for good *i* is still derived from the cost minimization problem and is given by $C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} C_t$ where $P_t = \left[\int P_{i,t}^{(1-\eta)} di\right]^{1/(1-\eta)}$. Unlike for deep habits, demand is not dynamic and firms face a more simple static pricing problem.

Relative Superficial Habit

In this specification, habits are also formed at the level of aggregate consumption good but, now, consumers get utility from the ratio of aggregate consumption to a measure of lagged consumption. Specifically, $x_t^j = \frac{C_t^j}{S_{t-1}^\theta}$. The overall demand for good *i* is given by $C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} C_t$ where $P_t = \left[\int P_{i,t}^{(1-\eta)} di\right]^{1/(1-\eta)}$. Under this specification, firms again have a static pricing problem.

2.6 Social Planner Problem

As a reference, we also consider the social planner solution to our model. The key difference between this and the flexible-price, competitive equilibrium is that the social planner internalizes the consumption externality and does not require a price markup over the costs of production. Furthermore, from the social planner perspective, there is no distinction between deep and superficial habits: the optimal process for aggregate habit S_t , implies an indentical process for each good-specific habit $S_{i,t}$. The social planner solves the utility maximization problem subject to the production function, resource constraint, and the process describing the evolution of the habit stock.

Formally, the planner solves,

$$\max_{\{x_t, h_t, C_t, S_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(x_t, h_t)$$
(31)

subject to

$$C_t = A_t h_t \tag{32}$$

$$x_t = C_t - \theta S_{t-1} \tag{33}$$

$$S_t = \rho S_{t-1} + (1-\rho)C_t \tag{34}$$

We let the $\lambda_{1,t}, \lambda_{2,t}$ and $\lambda_{3,t}$ be the lagrange multipliers corresponding to the constraints, respectively. The social planner equilibrium is then summarized by the following set of equations:

$$x_t = C_t - \theta S_{t-1} \tag{35}$$

$$S_t = \rho S_{t-1} + (1-\rho)C_t \tag{36}$$

$$C_t = A_t h_t \tag{37}$$

$$-\frac{U_{h,t}}{A_t} = [U_{x,t} + (1-\rho)\lambda_{3,t}]$$
(38)

$$\lambda_{3,t} = \beta E_t [-\theta U_{x,t+1} + \rho \lambda_{3,t+1}]$$
(39)

$$log(A_{t+1}/\bar{A}) = \rho_a log(A_t/\bar{A}) + \epsilon^A_{t+1}$$

$$\tag{40}$$

Equation 38 contains the additional third term, $(1 - \rho)\lambda_{3,t}$, which is not present in the corresponding decentralized equilibrium equation 19. Here, the social planner takes into account the negative effect on future utility of additional aggregate consumption today, since it raises the stock of habit, S_t . Equation 39 shows, in turn, how the shadow value of higher habit stock, S_t , evolves over time. This evolution has two components. The first term gives the expected effect on tomorrow's discounted marginal utility. The second term represents the shadow value's effect on the next period's shadow value.

For the special case of $\rho = 0$, the optimal allocations are derived from the following three equations and the exogenous process for the technology:

$$-\frac{U_{h,t}}{U_{x,t}} = A_t E_t \left[1 - \theta \beta \frac{U_{x,t+1}}{U_{x,t}} \right]$$
(41)

$$x_t = C_t - \theta C_{t-1} \tag{42}$$

$$C_t = A_t h_t \tag{43}$$

Equation 41 clearly shows that the social planner is concerned not only about the trade-off between consumption and leisure in any given period, but also the effects of today's consumption on tomorrow's utility. This interdependence gives rise to a time varying wedge between the social planner optimal real allocations and the flexible price decentralized equilibrium counterpart with markup subsidy (see equation 47).

3 Markups and Subsidies

Since we will find that endogenous markup variation drives our results for optimal policy, we first consider markups under a *flexible-price* decentralized equilibrium. Using the subsidies defined earlier, we can separate the effects of the consumption externality from the dynamics that can be directly attributed to good-specific habits. This analysis will be crucial to disentangling the roles of these two forces in generating higher optimal inflation volatility upon introducing sticky prices. For simplicity, in this section we assume that $\rho = 0$.

3.1 Markup Dynamics

Setting $\varphi = 0$ (flexible price decentralized equilibrium) in the symmetric equilibrium conditions 21-24 and rearranging (see appendix 6.2 for details), we find that:

$$\mu_t^A = (1 - 1/markup_t) + \theta \beta E_t \Omega_{t+1} \mu_{t+1}^A \tag{44}$$

where $\mu_t^A = 1/\epsilon_t^{SR}$ is the inverse of the short-run elasticity of demand and

$$\epsilon_t^{SR} = \eta_t \equiv \eta (1 - \theta \frac{C_{t-1}}{C_t}) \tag{45}$$

With additive-deep habits, ϵ_t^{SR} is time varying. Furthermore, short-run elasticity is always less than the constant long-run elasticity, given by η . This result is intuitive because a fixed habit stock (in the short run) makes it painful for the consumer to adjust her consumption levels in that good.

Combining equations 44-45, we get the following expression for the markup under additive-deep habit:

$$markup_t^A = \frac{\eta_t}{\eta_t - 1} \frac{1}{\left[1 + \theta \beta E_t \Omega_{t+1} \frac{\eta_t}{(\eta_t - 1)\eta_{t+1}}\right]}$$
(46)

The first term in equation 46 is clearly counter-cyclical, while the second term is typically pro-cyclical. However, in all the calibrations we have tried, markup is strongly counter-cyclical overall. Appendix 6.4 shows a similar derivation for relative habits. Table 1 summarizes the extent of markup-volatility under the various kinds of habit. For the superficial cases, markups are constant. The markup for the additivedeep case is highly variable compared to the relative-deep specification.

3.2 Steady State Markups

Evaluating equation 46 at the deterministic steady state, we get the following expression for the steady state markup,

$$markup_{ss}^{A} = \frac{\eta m}{\eta m - 1}$$

where $m = \frac{(1-\theta)}{(1-\theta\beta)}$. Note that m > 1 always. Table 2 compares the analytic steady-state markups for the no-habit, additive-deep, and relative-deep cases. In general, $markup_{ss}^R < markup_{ss}^{NH} < markup_{ss}^A$. Table 3 compares these values under the particular calibration described in section 4.1. While the relative-habit markup is significantly smaller than the no habit case, the additive habit markup represents only small increase compared to no habits.

3.3 Optimal Subsidy/Tax

With expressions for the markup, we can easily find the subsidies (taxes) that satisfy the definitions in section 2.4.

Proposition 1. The optimal subsidy rule, which implements the social planner's allocation as a flexibleprice decentralized equilibrium, is

$$\tau_t^{opt} = 1 - markup_t \frac{1}{\left(1 - \theta \beta \sum_{j=0}^{\infty} \beta^j \frac{A_t}{A_{t+1+j}}\right)}$$
(47)

where

$$markup_t = \frac{\eta_t}{\eta_t - 1} \frac{1}{\left[1 + \theta \beta E_t \Omega_{t+1} \frac{\eta_t}{(\eta_t - 1)\eta_{t+1}}\right]}$$
(48)

and

$$log(A_{t+j+1}/\bar{A}) = \rho_a log(A_{t+j}/\bar{A}) + \epsilon^A_{t+j+1}$$

$$\tag{49}$$

(Proved in the Appendix 6.5.1)

Expression 47 makes it clear that, under flexible prices, distortions in the economy can be reduced to a markup term and a productivity term. Once we introduce price rigidities into the environment, we will be able to distinguish which of these distortions is counter-balanced by monetary policy.

3.4 Markup Subsidy

Proposition 2. : The markup subsidy, which implements the perfect competition equilibrium as a flexibleprice decentralized equilibrium is :

$$\tau_t^{mkp} = 1 - markup_t = 1 - \frac{\eta_t}{\eta_t - 1} \frac{1}{[1 + \theta\beta E_t \Omega_{t+1} \frac{\eta_t}{(\eta_t - 1)\eta_{t+1}}]}$$
(50)

(Proved in the Appendix 6.5.2) This subsidy counter-acts only the markup distortion, leaving the inefficiency caused by the habit externality.

4 Optimal Policy

4.1 Ramsey Optimal Policy Problem

We study optimal policy from a "timeless perspective", following Woodford (2003). At time t = 0, the monetary authority is assumed to be operating for an infinite number of periods and in choosing the policy it is assumed to honor the past commitments. Then, monetary authority chooses the policy that maximizes households' welfare subject to the private sector competitive equilibrium conditions and $R_t \ge 1$, for the given exogenous process for technology. However, in solving the Ramsey problem, we assume that the zero lower bound on nominal interest rates can never bind.

4.1.1 Ramsey Equilibrium

Ramsey equilibrium is a set of processes $\Xi_{t=0}^{\infty}$ that maximizes

$$\max_{\{\Xi_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(x_t, h_t)$$
(51)

subject to the private sector symmetric competitive equilibrium conditions (18)-(28), given approcess for the exogenous technology shocks $\{A_t\}_{t=0}^{\infty}$, where

$$\Xi_t \equiv [C_t, h_t, y_t, \pi_t, R_t, x_t, \omega_t, \mu_t, \Omega_t, \lambda_t, \xi_t, \tau_t]_{t=0}^{\infty} \text{ and}$$
$$\Lambda_t \equiv [\lambda_{i,t}]_{t=(0,\infty)}^{i=(1:11)}$$

4.1.2 Calibration and Functional Forms

We impose the functional form $U(X,h) = \frac{X^{1-\sigma}}{1-\sigma} - \gamma h_t$, and consider the cases with and without subsidy, for the four "extreme" habit specifications [additive, relative]x[deep, superficial]. We solve the model using second-order approximation, following Schmitt-Grohé and Uribe (2004). We assume throughout that technology shocks are the only shocks in the economy.

We calibrate the model to U.S. economy following Ravn et al. (2006). We set the discount rate, β , so that the annual interest rate is 4 percent and normalize $\pi^* = 1$. The elasticity of substitution across varieties is assumed to be $\eta = 5.3$, and the risk aversion parameter is set to $\sigma = 2$. For additive habits (deep or superficial), we set the habit parameter to $\theta = 0.65$. For the relative habit cases, $\theta = -0.1$. The disutility of labor is fixed at $\gamma = 21$. We set the persistence parameters, ρ_a and ρ_s , to 0.85. The price stickiness parameter is set to $\varphi = 14$. Finally, for all versions of the model, we calibrate the standard deviation of the technology shock (σ_{ϵ_A}) so that the volatility of output is 0.015, roughly matching the postwar US data.

4.2 The Ramsey Steady-State

We study long-run state of the Ramsey equilibrium in an economy without uncertainty to characterize the optimal long-run inflation rate in a model with deep/additive habit formation along with nominal rigidities.

Proposition 3. : The optimum gross long run rate of inflation in the Ramsey steady state is one. That is

$$\bar{\pi} = 1 \tag{52}$$

(Proved in the Appendix 6.5.3)

Intuitively, higher average inflation has two opposing effects on markups. Given sticky prices, higher inflation today tends to decrease markups, while the *expectation* of high inflation tomorrow causes firms to increase their markup today. The proof in the appendix shows that these effects are exactly offsetting, leading to zero inflation in the non-stochastic steady state. This result holds for all the subsidy schemes we study in this paper.

4.3 Ramsey Optimal Stabilization Policy

In this section, we show that under certain habit specifications optimal policy entails significant inflation volatility, in contrast to models without habit. Two possible explanations for this are 1) time varying short run elasticity of demand (additive-deep habits) leading to markup variations and 2) the externality created by consumers ignoring the impact of their consumption on the aggregate habit stock. We show that it is primarily the first channel that gives rise to optimal inflation volatility.

4.3.1 With Markup Subsidy

We find that optimal inflation under the time-varying markup subsidy is zero for all habit specifications, echoing results from models without habit. Figure 1 displays the impulse responses of real and nominal model variables to a technology shock, for the model with additive-deep habit and the markup subsidy. The figure shows that ramsey optimal policy achieves the same real allocations as the decentralized, flexible price equilibrium of the model. The same result holds for the other three habit specifications as well.

Table 4 compares the unconditional welfare of agents under various policies, for the additive-deep and additive-superficial habit specifications. Note here that all results are based on linearization around the non-distorted steady state.¹ Under both deep and superficial habits, welfare with the markup subsidy equals that under the flexible price equilibrium. Due to time variation in the habit externality, welfare under ramsey policy is less than the welfare achieved by the social planner. Yet the markup subsidy is designed to offset only the inefficiency due to monopolistic markups. From this we conclude that the habit externality alone cannot account for the nonzero inflation volatility that we find in the model

¹In this approach, we use a constant subsidy to eliminate all distortions at the steady-state. For the markup subsidy, we further allow time-variation in the subsidy to offset markup variation induced by habit. We recenter only for the purposes of this table.

without subsidy.

4.3.2 Without Markup Subsidy

Figure 2 shows the impulse responses to a technology shock, for the model with superficial habits and no subsidy. Table 6 also summarizes relative volatility of inflation and the markup, for both specifications. Here, perfect price stability no longer holds. Yet, inflation volatility remains quite small, especially for the case of relative habits. Recall that with either additive or relative superficial habits, firms' desired markups are constant. Thus, the interaction of market power with the habit externality does create some incentive for the ramsey planner to tolerate non-zero inflation, even when desired markups are constant.

The lower-right quadrant of table 4compares the ramsey policy with constant subsidy to the constant inflation policy with the same subsidy, under superificial habits. Again, ramsey policy does not achieve the first-best allocation. However, it improves slightly on the constant inflation policy. A striking result is that ramsey policy without the markup subsidy does *better* than ramsey policy with the subsidy. The intuition for this can be found in equation 47. With a positive technology shock, the consumption externality (captured by the second term in the equation) become more severe; agents consume even more without considering their impact on habits. In contrast, the sticky-price friction tends to increase the markup, thereby decreasing consumption. By removing this offsetting force, the markup subsidy limits the ability of monetary policy to improve real allocations.

Figure 3 shows impulse responses for the model with deep habits, while table 7 provides a summary of relative inflation and markup volatility. We note two key differences between additive and relative deep habit. First, under relative habits, perfect price stability is the optimal policy, while with additive habits, optimal policy calls for significant inflation volatility. Recall that although firms' desired markup is varying endogenously in both specifications, time variation in the short run demand elasticity occurs only for the additive case (see equation 46). Again, table 1 shows that this second effect makes markups much more volatile under additive habits. Table 7 demonstrates that the second effect is the only one that monetary policy attempts to combat with non-zero inflation. In our model, therefore, the monetary authority optimally engages in a policy of *partial markup stabilization*, in contrast to the full-stabilization result in the no-habit model. Comparing these results to the case of superficial habits, we conclude that the optimality of nonzero inflation is primarily due to the desired markup variations arising from time varying short run demand elasticity.

The second key difference is the pro-cyclicality, on impact, of markups under additive habits. Ravn et al. (2006) show that, under flexible prices, deep habits of both kinds lead to counter-cyclical markups. Under optimal policy, this feature is preserved for relative habits, but figure 4 shows that is reversed for additive habits. After a technology shock, the monetary authority engages in a deflationary policy, leading to higher markups because firms cannot fully adjust their prices to achieve their desired markup. To achieve this, interest rates are initially slightly tighter, compared to a strong easing under the constant inflation policy (not shown).

The upper-right quadrant of table 4 compares the ramsey policy with constant subsidy to the constant inflation policy with the same subsidy for additive-deep habits. Again, neither achieves the first-best allocation. However, the difference in welfare between the two policies is much greater than anywhere else in the table. Compared to the superficial, optimal policy offers a much larger benefit over the zeroinflation policy. This result confirms our conjecture that the key goal of monetary policy is offsetting variation in firms' desired markups.

Finally, figure 5 shows optimal relative inflation volatility as a function of habit strength, θ , for additive deep habits with no subsidy. Optimal volatility is an increasing function of the habit parameter: as the habit strength is increased beyond the benchmark value, $\theta = 0.65$, optimal inflation volatility is also increasing. Intuitively, stronger habits increase the importance of variations in short run demand elasticity (see equation 46) and thus, increases optimal inflation volatility. Because estimates of habit strength vary, it is plausible that optimal policy calls for an even higher level of inflation volatility than we find here.

5 Conclusion

We have argued that optimal monetary policy design depends greatly on the "true" habit specification, yet we have not taken a stand on which specification is most reasonable. In ongoing research, we develop and estimate a more complex version of the model with capital accumulation and investment adjustment cost, which still nests the various habit specifications. Our early results suggest that the additive-deep specification is, indeed, the most plausible. If this is true, then the customer market feature will play an important role in model dynamics, and optimal monetary policy may look quite different than that suggested in earlier literature.

Table 1: Markup volatility under flexible prices for different habit specifications

	Superficial (Relative or Additive)	Additive Deep	Relative Deep
$\sigma_{markup}/\sigma_{Y}$	0.000	1.413	0.092

Table 2: Steady-state markup equations

Superficial/No Habit	Relative-Deep Habit	Additive-Deep Habit
$markup_{ss}^{NH} = \frac{\eta}{\eta - 1}$	$markup_{ss}^{R} = \frac{\eta}{\eta - 1} \frac{1}{1 - \theta\beta}$	$markup_{ss}^{A} = \frac{\eta m}{\eta m - 1}$

Table 3: Steady-state markup (baseline calibration)

Superficial/No Habit	Relative-Deep Habit	Additive-Deep Habit
$markup_{ss}^{NH} = 1.23$	$markup_{ss}^R = 1.13$	$markup_{ss}^A = 1.27$

Table 4: Unconditional welfare of agents under deep and superficial additive habit (Efficient Steady State = -1487.58)

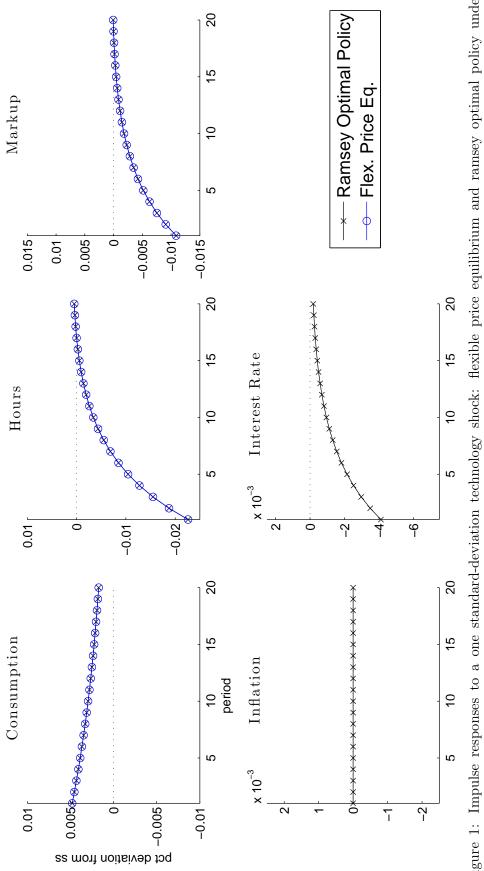
	Social Planner	Ramsey Policy = Flex Price Eq.	Ramsey Policy	Constant π Policy
		(Markup Subsidy)	(Constant Subsidy)	(Constant Subsidy)
Deep	-3.145	-3.682	-4.029	-5.699
Superficial	-3.145	-3.682	-3.620	-3.682

Table 5: Optimal inflation volatility under *deep habit* of different habit specifications, with markup subsidy.

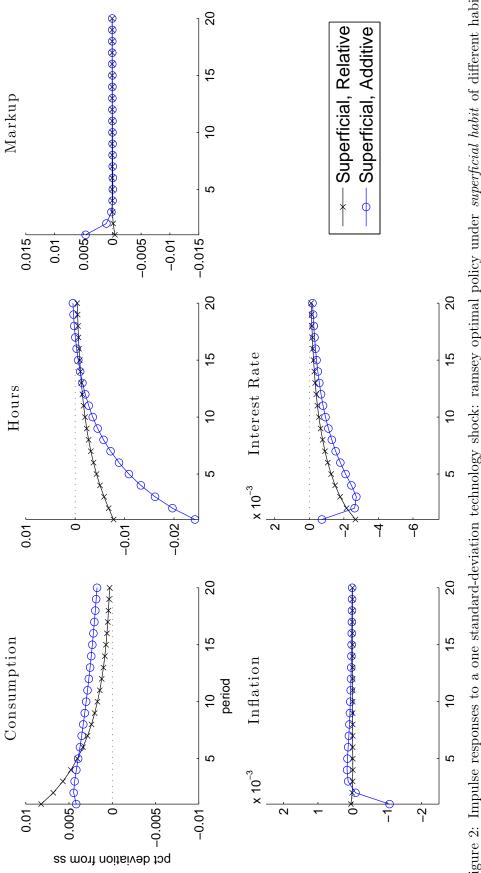
	Additive Habit	Relative Habit
σ_{π}/σ_{Y}	0.000	0.000
$\sigma_{markup}/\sigma_{Y}$	1.293	0.093

Table 6: Optimal inflation volatility under *superficial habit* of different habit specifications, without markup subsidy.

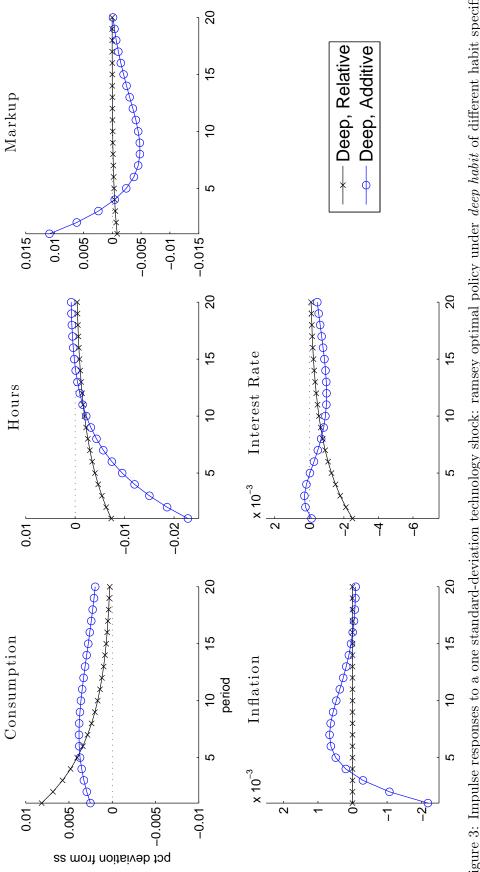
	Additive Habit	Relative Habit
σ_{π}/σ_{Y}	0.075	0.003
$\sigma_{markup}/\sigma_{Y}$	0.316	0.032













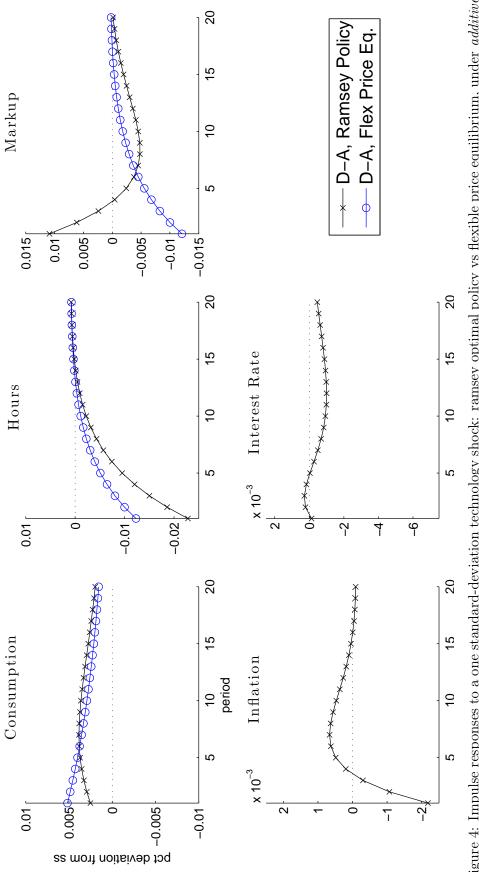




Table 7: Optimal inflation volatility under deep-habit of different habit specifications, without markup subsidy.

	Additive Habit	Relative Habit
σ_{π}/σ_{Y}	0.193	0.000
$\sigma_{markup}/\sigma_{Y}$	1.203	0.092

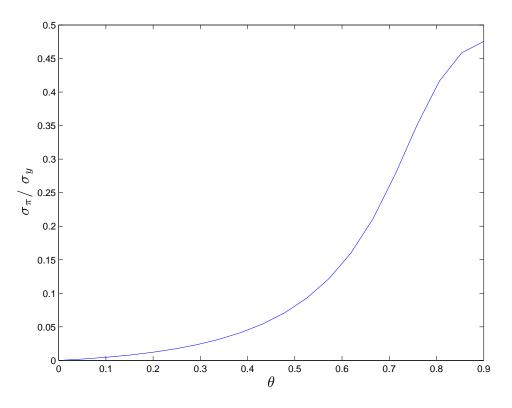


Figure 5: Optimal inflation volatility as a function of habit strength, θ , for *additive-deep* habits, with no subsidy.

6 Appendix

6.1 Steady State

In the deterministic steady state of the economy, the equilibrium conditions (19)-(30) reduce to the following system of equations:

$$x = c(1-\theta) \tag{53}$$

$$\tau = 0 \tag{54}$$

$$\omega = \gamma x^{\sigma} \tag{55}$$

$$\pi^* = \beta R \tag{56}$$

$$\mu = \frac{1}{\eta(1-\theta)} \tag{57}$$

$$\omega = \mu \left[\frac{\theta \beta (1-\rho)\right]}{1-\rho\beta} - 1\right] + 1 \tag{58}$$

$$C = h \tag{59}$$

$$y = h \tag{60}$$

$$S = C \tag{61}$$

$$\Omega = 1 \tag{62}$$

6.2 Markup Dynamics

We derive markup dynamics under additive-deep habit from the symmetric equilibrium conditions 21-24, for the special case $\rho = 0$. Setting $\varphi = 0$ in equation 21, we get the following relation between consumption, C_t and μ_t :

$$C_t = \eta \mu_t (C_t - \theta C_{t-1}) \tag{63}$$

which can be re-written as

$$\mu_t = \frac{1}{\eta(1 - \theta \frac{C_{t-1}}{C_t})}$$
(64)

Equations 22- 24 can be reduced to the following single equation

$$\mu_t = 1 - (1/markup_t) + \theta\beta E_t \Omega_{t+1} \mu_{t+1} \tag{65}$$

where $markup_t = 1/\lambda_t = A_t/\omega_t$.

Note that the short run price elasticity of demand is time varying and equal to the inverse of μ_t . Using the demand for good *i* derived from cost minimization problem:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} x_t + \theta C_{i,t-1}$$
(66)

we can find the short run price elasticity of demand as

$$\epsilon_{D,i,t}^{SR,P} = -\frac{\partial lnC_{i,t}}{\partial lnP_{i,t}} = \eta(1 - \theta \frac{C_{i,t-1}}{C_{i,t}})$$
(67)

Once we impose symmetric equilibrium, we see that, $\mu_t = 1/\epsilon_{D,t}^{SR,P}$.

Defining η_t as the short run price elasticity, we get an expression for the markup

$$markup_{t}^{A} = \frac{\eta_{t}}{\eta_{t} - 1} \frac{1}{\left[1 + \theta \beta E_{t} \Omega_{t+1} \frac{\eta_{t}}{(\eta_{t} - 1)\eta_{t+1}}\right]}$$
(68)

We also note that once we impose $\theta = 0$ into the markup equation, we get the constant markup expression in the baseline NK model.

6.3 Steady State Markup

Imposing the steady state conditions into 68, we get the expression

$$markup_{ss}^{AD} = \frac{\eta(1-\theta)}{\eta(1-\theta) - (1-\theta\beta)}$$
(69)

Dividing both numerator and denominator by $(1 - \theta\beta)$, we can simplify it as:

$$markup_{ss}^{AD} = \frac{\eta m}{\eta m-1}$$

where $m = \frac{1-\theta}{1-\theta\beta}$.

6.4 Relative Deep Habits

6.4.1 Symmetric Equilibrium Conditions

The symmetric equilibrium is governed by the following system of equations, where $w_t \equiv \frac{W_t}{P_t}$.

$$x_t = C/(S_{t-1}^{\theta}) \tag{70}$$

$$-\frac{U_h}{U_x} = \omega_t (1 - \tau_t) / S_{t-1}^{\theta}$$
(71)

$$E_t \left[\beta \frac{U_x(t+1)}{U_x(t)} \frac{s_{t-1}^{\theta}}{s_t^{\theta}} \frac{1}{\pi_{t+1}}\right] R_t = 1$$
(72)

$$\varphi \lambda_t (\pi_t - \pi_t^*) \pi_t = \varphi \beta E_t \Omega_{t+1} \lambda_{t+1} (\pi_{t+1} - \pi_{t+1}^*) \pi_{t+1} + (1 - \eta \mu_t) C_t$$
(73)

$$\mu_t = (1-\rho)\xi_t + 1 - \lambda_t \tag{74}$$

$$\xi_t = \beta E_t \Omega_{t+1} \left(\mu_{t+1} \frac{C_{t+1}}{S_t} \theta(1-\eta) + \rho(\xi_{t+1}) \right)$$
(75)

$$\lambda_t = \omega_t / A_t \tag{76}$$

$$C_t = A_t h_t - \frac{\varphi}{2} \left(\pi_t - \pi_t^* \right)^2$$
(77)

$$S_t = \rho S_{t-1} + (1-\rho)C_t \tag{78}$$

$$log(A_{t+1}/\bar{A}) = \rho_a log(A_t/\bar{A}) + \epsilon^A_{t+1}$$
(79)

$$\Omega_t \equiv \frac{U_x(x_t, h_t)}{U_x(x_{t-1}, h_{t-1})} = \left(\frac{x_t}{x_{t-1}}\right)^{-\sigma} \left(\frac{S_{t-1}}{S_{t-2}}\right)^{\theta}$$
(80)

6.4.2 Markup Dynamics

We derive markup dynamics with relative-deep habits from the symmetric equilibrium conditions 73-75, for the special case $\rho = 0$. Setting $\varphi = 0$ in equation 73, we get the following relation between consumption, C_t and μ_t :

$$C_t(1 - \eta \mu_t^R) = 0 \tag{81}$$

which implies that $\mu_t^R = 1/\eta$ is constant.

Equations 74-75 can be reduced to the following single equation

$$\mu_t^R = (1 - 1/markup_t^R) + \theta \beta E_t \Omega_t \mu_{t+1}^R (1 - \eta) \frac{C_{t+1}}{C_t}$$
(82)

where $\mu_t^R = 1/\epsilon_{D,t}^{SR,P} = 1/\eta$. Note that η is the price elasticity of demand under relative-deep habits.

Solving for the markup:

$$markup_t^R = \frac{\eta}{\eta - 1} \frac{1}{\left[1 - \theta \beta E_t \Omega_{t+1} \frac{C_{t+1}}{C_t}\right]}$$
(83)

The first term is the standard constant markup expression in NK models. The second term gives rise to time variation in markups.

6.5 Proofs of the Propositions

6.5.1 Proof of Proposition 1 (special case $\rho = 0$)

Social planner's equilibrium conditions (from section 3.7.):

$$x_t = C_t - \theta C_{t-1} \tag{84}$$

$$C_t = A_t h_t \tag{85}$$

$$-\frac{U_{h,t}}{A_t} = [U_{x,t} + \lambda_{3,t}]$$
(86)

$$\lambda_{3,t} = -\theta\beta E_t[U_{x,t+1}] \tag{87}$$

Flexible price (imperfectly) competitive equilibrium conditions are, on the other hand, given by the following set of equations (from section 3.4., once we set $\varphi = 0$):

$$x_t = C_t - \theta C_{t-1} \tag{88}$$

$$C_t = A_t h_t \tag{89}$$

$$-\frac{U_{h,t}}{U_{x,t}} = \omega_t (1 - \tau_t) \tag{90}$$

$$\mu_t = \frac{1}{\eta(1 - \theta \frac{C_{t-1}}{C_t})}$$
(91)

$$\mu_t = \theta \beta E_t \Omega_{t+1} \mu_{t+1} + 1 - \lambda_t \tag{92}$$

$$\lambda_t = \frac{\omega_t}{A_t} \tag{93}$$

$$markup_t = \frac{1}{\lambda_t} \tag{94}$$

where $U_{h,t} = \gamma$ and $U_{x,t} = x_t^{\sigma} = (C_t - \theta C_{t-1})^{\sigma}$.

Our approach to calculate optimal subsidy/tax is as follows: we first solve the social planner's problem and characterize pareto optimal allocation. Then, we calculate the optimal subsidy/tax that support this allocation as a decentralized equilibrium.

Substituting the social planner's equilibrium condition 86 into 87 and solving recursively, we can write shadow value of habit stock, $\lambda_{3,t}$ as the following:

$$\lambda_{3,t} = \theta \beta \gamma E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{1}{A_{t+j+1}} \right]$$
(95)

Substituting $\lambda_{3,t}$ in social planner equilibrium condition 86, we can express the optimal consumption path with the following equation:

$$(C_t^{opt} - \theta C_{t-1})^{-\sigma} = \frac{\gamma}{A_t} [1 - \theta \beta E_t \sum_{j=0}^{\infty} \beta^j \frac{A_t}{A_{t+j+1}}]$$
(96)

Imposing expression 96 into the decentralized (imperfectly) competitive equilibrium condition 90, we then find the optimal labor subsidy/tax :

$$\frac{1}{[1 - \theta \beta E_t \sum_{j=0}^{\infty} \beta^j \frac{A_t}{A_{t+j+1}}]} = \frac{1}{markup_t} (1 - \tau_t^{opt})$$
(97)

Rearranging, we find that

$$\tau_t^{opt} = 1 - markup_t \frac{1}{[1 - \theta \beta E_t \sum_{j=0}^{\infty} \beta^j \frac{A_t}{A_{t+j+1}}]}$$
(98)

where

$$markup_t = \frac{\eta_t}{\eta_t - 1} \frac{1}{\left[1 + \theta \beta E_t \Omega_t \frac{\eta_t}{(\eta_t - 1)\eta_{t+1}}\right]}$$
(99)

and

$$log(A_{t+j+1}/A_{bar}) = \rho_a log(A_{t+j}/A_{bar}) + \epsilon^A_{t+j}$$
(100)

6.5.2 Proof of Proposition 2

The flexible price perfectly competitive equilibrium is given by the following set of equations:

$$-\frac{U_{h,t}}{U_{x,t}} = A_t \tag{101}$$

$$x_t = C_t - \theta C_{t-1} \tag{102}$$

$$C_t = A_t h_t \tag{103}$$

Government subsidy is used only to correct the markup inefficiency (habit externality is not taken into account while we determine the markup subsidy). Comparing 90to 101, we find that $\tau_t^{mkp} = 1 - \frac{A_t}{\omega_t}$. The result will follow by substituting the markup expression 68.

6.5.3 Proof of Proposition 3

First order condition of the Ramsey problem associated with inflation:

$$\lambda_{3,t-1}(\Omega_t/\pi_t^2) - \varphi \frac{\omega_t}{A_t} (\lambda_{4,t} - \lambda_{4,t-1}\Omega_t)(2\pi_t - 1) + \varphi \lambda_{8,t}(\pi_t - 1) = 0$$
(104)

The first order condition for R_t implies that $\lambda_{3,t} = 0$ for all t. Furthermore, $\Omega = 1$ in the deterministic steady state of the model, implying that the second term is also zero. The first-order condition with respect to hours shows that $\lambda_{8,t} = -\frac{\gamma}{A_t} < 0$ for all t. Since under sticky prices $\varphi \neq 0$, it follows that $\bar{\pi} = 1$. This proof holds, regardless of the subsidy process.

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