Frequently Asked Questions Regarding
"Recoverability and Expectations-Driven Fluctuations"

by Ryan Chahour and Kyle Jurado

1. *Don't structural disturbances always need to be unpredictable?*

   No. Structural disturbances represent exogenous random changes (impulses) to the economic system, but they do not need to be unpredictable. From a theoretical perspective, it is always possible to represent agents' information structure equivalently in terms of purely unpredictable exogenous disturbances (e.g. see the "news representation" in Theorem 1 of Chahrour and Jurado (2018). However, these disturbances carry different economic interpretations and are identified using different theoretical restrictions. We discuss this issue in more detail starting in the second paragraph of Section (5) in the paper, and explain why the unforecastable disturbances are not appropriate for our application.

2. *Isn't the recoverability condition virtually always satisfied when the number of observables equals the number of disturbances?*

   It is true that recoverability can often be satisfied even when stronger conditions like invertibility are not. This is a main advantage of focusing on recoverability; it expands the set of disturbances for which econometric strategies like VAR analysis are applicable.

   But it is also possible that some observables contain redundant information relative to the others. We construct such a case in Example 1 of Section (2) of the paper, but recoverability also fails in some well-known empirical models, including Barsky and Sims (2012) and Blanchard et al. (2013). This is why it can be important to check condition in Theorem (1) in practice.

3. *If a structural disturbance is recoverable given a certain set of observables according to my candidate theoretical model(s), how do I decide which structural restrictions to impose to identify this disturbance in a VAR analysis?*

   Knowing that a disturbance is recoverable does not tell us how to recover it. It just says that, according to the relevant economic theory, the set of observables under consideration contain enough information to identify the disturbance. As in the structural VAR literature that presumes invertibility, selecting the appropriate subset of restrictions to impose is a matter of economic judgment, which will vary depending on the context. In Section (5) we present a set of restrictions that can be used in the context of identifying technological disturbances in the presence of advance information, even if those disturbances are non-causal or non-invertible.

4. *Can I use Theorem (1) to test recoverability in the data?*

   No. Like existing tests for invertibility (e.g. Fernández-Villaverde et al., 2007), recoverability is a property of the disturbances of a structural model and can only be tested given a (set of) candidate model(s). Articulating the set of candidate models remains an important, and challenging, step in all structural VAR analysis.

   Tests of theoretical recoverability and invertibility are distinct from empirical tests of "informational sufficiency" (e.g. Forni and Gambetti, 2014). These tests ask whether a given set of historical time series span all of the information actually available to people in the economy. We discuss this distinction in Section (4), in the paragraph starting with "Third, ..."

5. *Are you saying that I always need a fully-specified theoretical model in order to use techniques like structural VAR analysis?*

   No. The spirit of our recoverability test, like existing tests of invertibility (e.g. Fernández-Villaverde et al., 2007), is the spirit of a Monte Carlo test of an empirical procedure: propose a theoretical data generating process that the researcher believes is representative of actual data, and then use the theoretical data generating process to examine some property of the empirical procedure. Much of the VAR literature assumes invertibility without proving that it holds for any particular data generating processes. The benefit of focusing on recoverability, instead of invertibility, is that it expands the set of data generating processes for which VAR strategies may be valid.

6. *Is recoverability a necessary condition for identifying impulse responses or variance shares?*

No. Recoverability is about determining when the structural disturbances are identified. Of course, if the disturbances are identified then other objects of interest like impulse responses and variance shares are also identified; however, the converse is not true. We discuss this in the last paragraph of Section (4) .

7. *Isn't recoverability unhelpful in situations when we are interested in forecasting or estimating disturbances that occurred towards the end of the sample period?*

   Forecasting does not require the identification of structural disturbances; representations with reduced-form disturbances (i.e. the Wold representation) are sufficient.

   Regarding estimating disturbances toward the end of the sample, it is possible for standard errors to be large for disturbances that are identified mainly by observables at later dates. This is analogous to how standard errors can be large for disturbances at the beginning of the sample when those disturbances are identified mainly by observables at earlier dates. Recoverability is about identification, not about inference. Obviously both are important, but in this paper we focus on the first.

8. *In your empirical application, you assume technology is driven by a single shock. What happens if technology is driven by several disturbances, and only a subset of these can be imperfectly observed in advance?*

   Information structures of the latter sort, sometimes referred to as "noisy-news", are observationally equivalent to information structures with a pure "noise representation", in which productivity is driven by a single disturbance and signals depend on all of the disturbances to productivity. Moreover, the shocks in a noisy-news information structure are typically not recoverable while the disturbances in a noise representation are. We discuss these issues in Section (5) of the paper and the representation result is proved in Chahrour and Jurado (2018).

9. *Why don't the identification restrictions you use in your application fit within the framework of Lippi and Reichlin (1994)?*

   Lippi and Reichlin (1994) restrict attention to causal representations, but we do not. We clarify this the fifth paragraph of the Introduction. One point of our paper is to argue that this is not a necessary condition for using VARs. Our application presents one case in which it would be inappropriate to consider only causal representations. This identification procedure makes no assumption about the causality (or not) of the underlying economic model. The fact that we cannot reject the null hypothesis of non-causality in the empirical application demonstrates why relaxing this assumption is important.

If only causal structural disturbances are allowed, then it is possible to show that the vector of reduced-form VAR residuals can always be expressed as a linear transformation of the structural disturbances, where that linear transformation can be represented by a Blaschke matrix. However, without imposing causality this is no longer true. Instead, all that can be said about the transformation is that it is unitary.

One source of confusion on this point could be a particular statement in Lippi and Reichlin's paper. They say: "if u_t is orthonormal white noise, then $A(L)$ is a BM [Blaschke Matrix] if and only if the vector $v_t = A(L)u_t$ is also an orthonormal white noise" (p. 311). In order to be true, this claim needs to be modified; either (i) the "if" part of the "if and only if" statement should be removed, or (ii) "Blaschke Matrix" should be replaced by "unitary matrix."

In our view, the specific type of transformation relating structural and reduced-form disturbances is not economically relevant, and focusing on the class of such transformations can misleadingly suggest that more economic restrictions are required. We discuss this in Section (4), in the paragraph starting with "In addition ..."

10. *Instead of using the condition in Theorem (1), can't I check recoverability by (i) simulating long samples of data, regressing the realized structural disturbances on many leads and lags of the observables, and checking whether the residuals are close to zero? Or, by (ii) using the Kalman smoother to compute the prediction error variance associated with the smoothed estimate and check whether it is zero?*

    Yes, in principle, though these approaches are unlikely to work as well in practice. The idea behind our proof of Theorem (1) is to perform these exercises in population for any linear model, and then determine what properties the model needs to satisfy for the prediction error to be zero. In practice, however, the simulation step in procedure (i) involves approximation and therefore introduces unnecessary numerical error. It is also more time consuming. Procedure (ii) involves using the Kalman smoother, which is only applicable to models with a state-space structure; therefore it is less general. It also involves solving a discrete algebraic riccati equation which introduces additional numerical error. The condition in Theorem (1) checks recoverability more directly and efficiently compared to these conceptually correct but more indirect approaches.

11. *Regarding your motivating example in Section (3), can't we always rewrite a non-causal signal in causal form? For example, if $s_t = \varsigma_1 \varepsilon^a_{t+1} + v_t$ couldn't we define $\tilde{\varepsilon}^a_t \equiv \varepsilon^a_{t+1}$ and write $s_t = \varsigma_1 \tilde{\varepsilon}^a_t + v_t$, which is causal with respect to $\{\tilde{\varepsilon}^a_t\}$?*

    Yes, *given* a signal structure, we can rewrite it in causal form. But this does not mean that it is without loss of generality to consider only causal signals. First, we do not know the signal structure a priori — that is partly what we are trying to learn from the empirical exercise. Contrary to the example above, we would not know a priori whether the signal depended only on ε^a_{t+1}, or also on other elements in the sequence $\{\varepsilon^a_{t+1}, \varepsilon^a_{t+2}, \dots\}$. The identification restrictions we propose have the advantage that they do not require this knowledge; we discuss this in Section (5), in the paragraph starting with "Third, ..."

    Second, the interpretation of the disturbances in the causal representation is different. The transformed disturbances cannot be interpreted as contemporaneous random changes in technology; in the example above, $\tilde{\varepsilon}^a_t$ represents the random change in technology that occurs at time $t+1$. With more complicated signals, e.g. $s_t = (1 - \rho L^{-1})^{-1}\varepsilon_t + v_t$, the interpretation becomes complicated further. Because our purpose in the application is to analyze the effects of physical technological disturbances, which do not depend on assumptions about the type of advance information agents might have, causal disturbances like $\tilde{\varepsilon}^a_t$ are not appropriate.

12. *Since computable numbers are measure zero, isn't it possible that the numerical procedure to check recoverability you describe could deliver a "false negative," which would lead you to reject recoverability numerically even though the disturbance is actually recoverable?*

Yes, but this is not relevant for practical purposes. By linear regularity, the value of the expression in Theorem (1) is the same for almost all values of λ on $\Delta = [-\pi, \pi]$. Therefore, the probability of getting a false negative for some value of λ randomly drawn in this interval is zero. It is true that, strictly speaking, it is not possible for a computer to draw λ randomly; it can only approximate a random draw using a pseudo-random number generator. But we should only worry about false negatives to the extent that we worry about pseudo-random numbers being poor approximations to truly random numbers.

Furthermore, in the common case that that linear mapping in equation (1) of the paper has a state space structure we can avoid the issue of random number generation. This is because the measure zero set $\Delta^* \subset \Delta$ on which the value of the expression in Theorem (1) differs from its value almost everywhere on Δ is finite and can be explicitly computed. The condition in Theorem (1) can then be checked for any value $\lambda \in \Delta \setminus \Delta^*$. The following Matlab program computes the finite set Δ^* for state-space models . It calls the function `tzero.m` which is part of Matlab's Control System Toolbox.

```
function [lc] = lamcrit(A,B,C)
% -------------------------------------------------------------------
% Find critical values of lambda for which the expression in Theorem 1
% takes on a different value from its value almost everywhere on
% [-pi,pi], for state-space models of the form:  y(t) = A*x(t) and
% x(t) = B*x(t-1) + C*e(t), e(t)~WN(0,I)
% -------------------------------------------------------------------
nx  = size(B,1);
ny  = size(A,1);
ne  = size(C,2);
iz  = tzero(eye(nx),C,-A,zeros(ny,ne),B);
tol = 1e-10;
ind = 1;
tz  = [];
for i = 1:length(iz)
    H = A/(eye(nx)-B.*iz(i))*C;
    r = rank(H,tol);
```

```
        if r < ne
            tz(ind) = iz(i);
            ind = ind + 1;
        end
    end
    lc = 1i.*log(tz);
    lc = lc(abs(imag(lc))<tol);
    end
```

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