

# Anticipated Productivity and the Labor Market

## ONLINE APPENDIX

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### I Nash Bargaining

Nash bargaining is a common paradigm for wage determination in models of random matching. To investigate how well the model performs under this version of the wage, we solve for the Nash bargained wage implied by our model and then re-estimate the model.

#### I.1 Solution

The Nash bargained wage satisfies

$$W_t^{NB} = \arg \max_{W_t} [\bar{\mathbf{W}}_t(W_t) - \bar{\mathbf{U}}_t]^\eta [\mathbf{J}_t(W_t) - \mathbf{V}_t]^{1-\eta}, \quad (\text{I.1})$$

where  $\bar{\mathbf{W}}_t$  denotes the value of a match for the household,  $\bar{\mathbf{U}}_t$  denotes the value of unemployment for the household,  $\mathbf{J}_t$  denotes the value of a match for the firm, and  $\mathbf{V}_t$  denotes the value of a vacancy for the firm. Free-entry of firms implies that  $\mathbf{V}_t = 0$ , and our specification of unemployment benefits, combined with the existence of a participation margin for households, implies that  $\bar{\mathbf{U}}_t = \kappa_t$ . Thus, the Nash sharing rule reduces to

$$\bar{\mathbf{W}}_t - \bar{\mathbf{U}}_t = \left( \frac{\eta}{1-\eta} \right) \mathbf{J}_t. \quad (\text{I.2})$$

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The household match surplus (in units of consumption) may be expressed as the sum of the wage payment earned in the period of the match (due to our timing assumption) and the continuation value of the match, less the lump-sum transfer to the unemployed,

$$\bar{\mathbf{W}}_t - \bar{\mathbf{U}}_t = W_t - \kappa_t + (1 - \lambda)E_t \left\{ (1 - p_{t+1})\Omega_{t,t+1}(\bar{\mathbf{W}}_{t+1} - \bar{\mathbf{U}}_{t+1}) \right\}. \quad (\text{I.3})$$

The value of a match to the firm (again, in units of consumption) is given by the current marginal product of the match net of the wage bill plus the continuation value,

$$\mathbf{J}_t = F_{N,t} - W_t + (1 - \lambda)E_t \left\{ \Omega_{t,t+1}\mathbf{J}_{t+1} \right\}. \quad (\text{I.4})$$

To solve for the wage associated with Nash bargaining, begin by substituting the expressions for  $\bar{\mathbf{W}}_t$  and  $\bar{\mathbf{U}}_t$  into the Nash sharing rule,

$$W_t^{NB} - \kappa_t + (1 - \lambda)E_t \left\{ (1 - p_{t+1})\Omega_{t,t+1}(\bar{\mathbf{W}}_{t+1} - \bar{\mathbf{U}}_{t+1}) \right\} = \frac{\eta}{1 - \eta}\mathbf{J}_t. \quad (\text{I.5})$$

Iterating the sharing rule forward and substituting in for  $\bar{\mathbf{W}}_{t+1} - \bar{\mathbf{U}}_{t+1}$ ,

$$W_t^{NB} - \kappa_t + (1 - \lambda)E_t \left\{ (1 - p_{t+1})\Omega_{t,t+1} \left( \frac{\eta}{1 - \eta} \right) \mathbf{J}_{t+1} \right\} = \frac{\eta}{1 - \eta}\mathbf{J}_t. \quad (\text{I.6})$$

Replacing  $\mathbf{J}_t$  with the firm's first-order condition for labor and using  $\mathbf{J}_{t+1} = \phi_{t+1}^N$ ,

$$\begin{aligned} W_t^{NB} - \kappa_t + (1 - \lambda)E_t \left\{ (1 - p_{t+1})\Omega_{t,t+1} \left( \frac{\eta}{1 - \eta} \right) \phi_{t+1}^N \right\} \\ = \frac{\eta}{1 - \eta} (F_{N,t} - W_t^{NB} + (1 - \lambda)E_t \left\{ \Omega_{t,t+1}\phi_{t+1}^N \right\}). \end{aligned} \quad (\text{I.7})$$

Solving for  $W_t^{NB}$ , we obtain

$$W_t^{NB} = (1 - \eta)\kappa_t + \eta \left[ F_{N,t} + (1 - \lambda)E_t \left\{ \Omega_{t,t+1}p_{t+1}\phi_{t+1}^N \right\} \right]. \quad (\text{I.8})$$

The stationary representation used for estimation is obtained by dividing through by  $X_{t-1}$ , which yields

$$\tilde{W}_t^{NB} = (1 - \eta)\tilde{\kappa}_t + \eta \left[ \tilde{F}_{N,t} + (1 - \lambda)\gamma_{x,t}E_t \left\{ \Omega_{t,t+1}p_{t+1}\tilde{\phi}_{t+1}^N \right\} \right]. \quad (\text{I.9})$$

Table I.1: Parameter Estimates (Nash bargaining)

Parameter	Concept	Estimate
$\theta$	Labor supply elasticity	10.000
$\sigma$	Inv. intertemporal elasticity	0.500
$\xi$	Vac. posting cost (curvature)	0.056
$\epsilon$	Matching function elasticity	0.950
$\omega^{NB}$	Nash Term	0.500

## I.2 Calibration

Our calibration strategy, described in Section 4 and Appendix B, pins down all endogenous variables and parameters in the steady state version of (I.9), except for  $\eta$ , the bargaining share parameter. Accordingly, to ensure that our long-run restrictions are satisfied, we choose  $\eta$  to solve (I.9), given the steady state values we compute above:

$$\eta = \frac{(1 - \kappa)}{\frac{F_n + (1 - \lambda)\gamma_x \Omega p \gamma_x a_n / q}{W} - \kappa}. \quad (\text{I.10})$$

## I.3 Estimation and results

We estimate the model under Nash bargaining in the same way we estimate the model under the flow wage. In particular, we allow the data to choose between the model and a simple inertial wage rule:

$$W_t = (W_t^{NB})^{\omega^{NB}} W_{t-1}^{1 - \omega^{NB}}. \quad (\text{I.11})$$

Table I.1 reports the parameter estimates from our estimation of the model with Nash bargaining. We immediately see that the parameter estimates are all hitting their bounds with the exception of  $\xi$ .<sup>1</sup> Most notably,  $\omega^{NB}$ —the parameter that governs the relative strengths of the inertial and Nash components of the wage in (I.11)—is at its lower bound of 0.5. This indicates that the data unambiguously prefer an inertial wage to the Nash bargained wage. Put differently, the model with Nash bargaining would perform even worse if we were to impose  $\omega^{NB} = 1$ , thus insisting that Nash bargaining hold exactly.

Figure I.1 reports the empirical and model-based impulse responses to our identified shock. Not surprisingly in light of the results in Table I.1, the model with Nash bargaining

<sup>1</sup>Because our point estimates are all hitting bounds, the corresponding standard errors are invalid, so we do not report them.

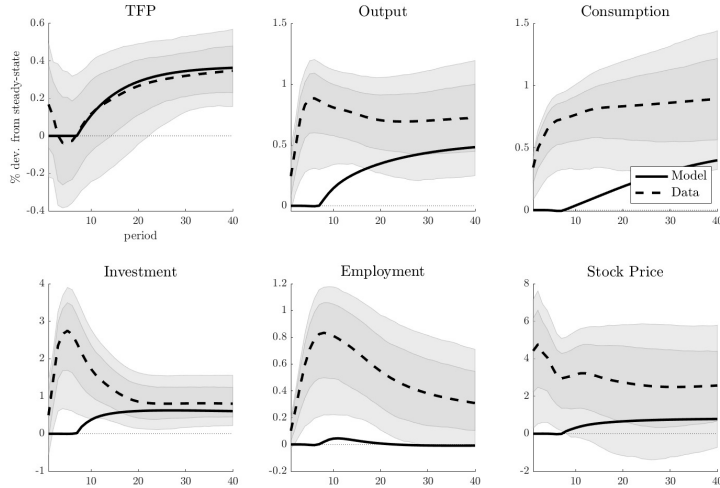


Figure I.1: Estimated model with Nash bargained wage

cannot generate the magnitude of responses that we observe in the data, especially in the period of anticipation. In fact, output, consumption and investment each *fall* during the anticipation period, whereas all three series rise strongly in the data.

As we discuss in the text, the model with Nash bargaining is unable to account for the data because the Nash-bargained wage is fundamentally forward-looking: A boom in employment and consumption today will increase the present discounted value of a match, thus driving up the Nash bargained wage and precluding the original rise in employment and consumption. This negative feedback thus chokes off any substantial response under Nash bargaining during the anticipation period, a result which bears out in Figure I.1.

## II Data Sources and Construction

Our main VAR specification consists of TFP, output, consumption, investment, employment, and the stock price. Except when otherwise noted, we download these series from the FRED database of the St. Louis Federal Reserve Bank.

For TFP, we use the capacity utilization adjusted measure described by [Basu et al. \(2006\)](#) and downloaded from <https://www.frbsf.org/economic-research/indicators-data/> on September 5, 2019. To compute the level of TFP we cumulate the growth rates starting from the initial observation in 1947Q2.

Quantity variables are provided in real per-capita terms. Our population series is the

civilian non-institutional population ages 16 and over, produced by the BLS. We convert our population series to quarterly frequency using a three-month average and smooth it using an HP-filter with penalty parameter  $\lambda = 1600$  to account for occasional jumps in the series that occur after census years and CPS rebasings (see [Edge and Gürkaynak \(2010\)](#)). Our deflator series is the GDP deflator produced by the BEA national accounts.

For output, we use seasonally adjusted nominal output produced by the BEA divided by the population and the GDP deflator. For investment, we take the sum of nominal gross private domestic investment and personal expenditures on durable goods, again divided by the population and the GDP deflator. Consumption consists of nominal personal consumption expenditures on non-durables and services, also divided by the GDP deflator and population. Our measure of employment is total non-farm payroll employment from the BLS’s Current Establishment Survey (CES) and is also divided by the population. Lastly, our measure of real stock prices is based on the NYSE index from the Center for Research in Security Prices (CRSP) and is deflated by the GDP deflator and divided by the population.

Our set of auxiliary variables  $\mathbf{W}_t$  includes 19 measures of aggregate and sectoral wages. Our preferred wage measure comes from the BEA National Accounts, series code A132RC, and consists of wage and salary compensation for private industries. To arrive at an hourly wage, we divide this by total private sector hours from the BLS Labor Productivity and Costs release (Nonfarm Business Sector: Hours of All Persons) and the GDP deflator.

The additional elements of the wage panel include: (i) median weekly earnings divided by the GDP deflator from the BLS’s Current Population Survey, (ii) the new hire real wage series produced by [Basu and House \(2016\)](#) and downloaded from <https://www.nber.org/data-appendix/w22279/>, and (iii) sixteen additional hourly wage series originating from the super-sector classification level of the CES. These series are listed in [Table II.2](#). We download each from the FRED database in nominal terms and then divide by the GDP deflator to arrive at real hourly wages.

Other labor market responses are constructed by adding a set of standard series to  $\mathbf{W}_t$ . The vacancies series is taken from [Barnichon \(2010\)](#), which splices together measures of print and online help-wanted advertising. Labor force participation is the Civilian Labor Force Level, produced by the BLS, divided by the same population series used to construct our other per-capita measures. Our hours series is the BLS’s Hours Worked for All Employed Persons in the Nonfarm Business Sector. The unemployment series is the standard measure constructed by the BLS.

Table II.2: CES Sectoral Wage Series

<b>Sector</b>	<b>Code</b>
Total Private	AHETPI
Goods Producing	CES0600000008
Mining	CES1000000008
Manufacturing	CES3000000008
Services	CES0800000008
Trade, Transportation, and Utilities	CES4000000008
Wholesale Trade	CES4142000008
Retail Trade	CES4200000008
Transportation and Warehousing	CES4300000008
Utilities	CES4422000008
Information	CES5000000008
Financial Activities	CES5500000008
Professional and Business Services	CES6000000008
Education and Health Services	CES6500000008
Leisure and Hospitality	CES7000000008
Other Services	CES8000000008

Finally, we consider two measures of the job-finding probability. The first is based on monthly unemployment data, and is constructed as

$$JFP_t^1 \equiv \frac{U_{t-1} - (U_t - U_t^{st})}{U_{t-1}} \quad (\text{II.1})$$

where  $U_t$  is the total number of unemployed workers in period  $t$  and  $U_t^{st}$  is the total number of short-term (less than 5 weeks) unemployed workers. We construct the monthly series for  $JFP_t^1$ , and then compound the monthly probabilities over three months to get quarterly job-finding probabilities. Our second job-finding probability series is based on the JOLTS survey, and only exists for the post-2000 sample. We construct it as

$$JFP_t^2 \equiv \frac{NH_t}{U_t + NH_t} \quad (\text{II.2})$$

where  $NH_t$  is the gross number of newly hired workers. The timing in this formula is designed to be consistent with our assumption that workers begin work in the same period they are hired.

Table III.3: Variance decomposition of VAR variables (time domain)

Horizon	TFP	Y	C	I	N	NYSE
0	0.06	0.17	0.69	0.06	0.22	0.30
4	0.02	0.62	0.86	0.55	0.61	0.35
8	0.02	0.72	0.91	0.57	0.66	0.36
12	0.04	0.78	0.93	0.59	0.68	0.38
16	0.07	0.82	0.94	0.60	0.69	0.39
20	0.12	0.84	0.95	0.61	0.70	0.39
40	0.36	0.88	0.95	0.65	0.67	0.41
80	0.63	0.91	0.95	0.69	0.64	0.53
200	0.78	0.90	0.92	0.71	0.62	0.67

### III Additional Results and Robustness

#### III.1 Variance Decomposition

Table III.3 reports the variance decomposition of our identified shock in the time domain.

The identified shock explains over 60% of both output and employment at short horizons (by one year), and explains at least this much of both variables at all longer horizons. On the other hand, the shock only explains a small fraction of TFP (less than 10%) at horizons under five years, but thereafter explains an increasingly large fraction of TFP, ultimately growing to nearly 80%. These patterns are consistent with the notion of “technological diffusion news” that our procedure is designed to identify and indeed are similar to the results in Portier (2015).

#### III.2 Empirical Exercise

Our empirical impulse responses are robust to (i) changing the number of lags in the VAR, (ii) running a VECM imposing one, two, or more trends in the data, (iii) expanding the set of observables in  $\mathbf{Y}_t$  to include additional variables, such as alternative labor market indicators, and (iv) changing the sample period used for estimation.

For example, restricting the sample to start in 1985—a common alternative start date in the VAR literature—delivers qualitatively similar responses for all variables. We plot these responses in Figure III.2.

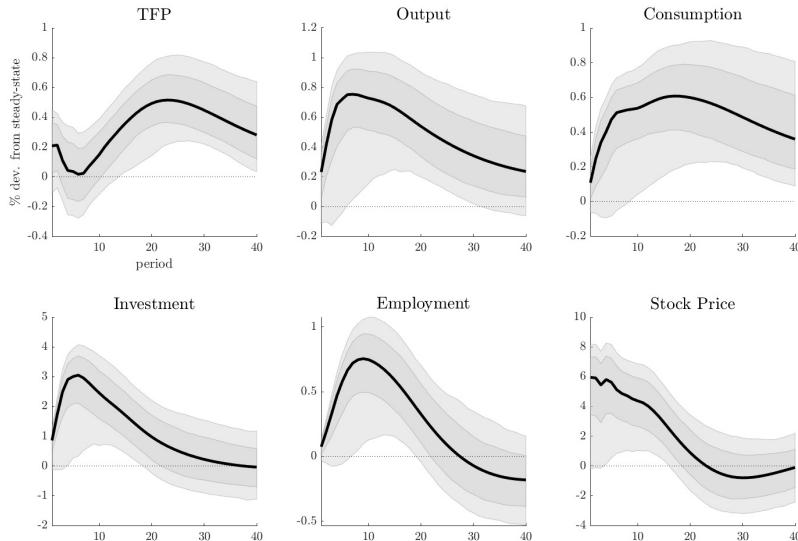


Figure III.2: VAR impulse responses for the short sample starting in 1985Q1

### III.3 Inflation and Nominal Wages

While the model we study is entirely real, news shocks are often estimated to induce a fall in inflation (our empirical analysis is consistent with this observation—see Figure 3). This leads to two questions: First, if we were to consider a model with nominal rigidities, would a news shock lead to a fall in inflation as in the data? Second, given the observed fall in inflation and real wages following our identified shock, do our results imply that news shocks lead to a decline in the level of nominal wages (that would be difficult to square with the data, in which average nominal wages rarely decline)?

While fully spelling out a version of our model with nominal rigidities is beyond the scope of this paper, it is nevertheless possible to assess the predictions that such a model would likely make for how inflation responds to a news shock—and to determine whether that response is qualitatively similar to what we find in Figure 3. To do this, observe that in a broad class of sticky-price models, inflation dynamics are determined by an expression of the form (Sbordone, 2002; Barsky et al., 2015):

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \left[ \frac{(1 - \beta)(1 - \beta\zeta)}{\zeta} \right] \hat{MC}_t \quad (\text{III.1})$$

where  $\zeta$  is a parameter that governs the degree of price stickiness,  $MC_t$  is a firm’s real marginal cost,  $\pi_t$  is inflation, and hats indicate percent deviations from steady-state values.



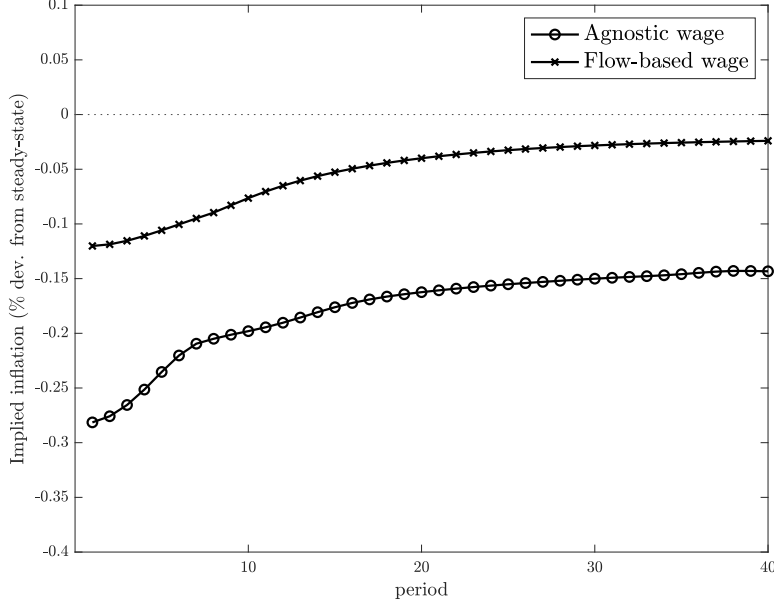


Figure III.3: Implied inflation responses

Considering for simplicity the limit in which vacancy posting costs become small, our model implies:<sup>2</sup>

$$\hat{M}C_t = (1 - \alpha)(\hat{W}_t - \hat{\gamma}_{x,t}) + \alpha\hat{R}_t \quad (\text{III.2})$$

where, using the notation from Appendix A,  $\tilde{W}_t \equiv W_t/X_{t-1}$  is the wage stationarized with respect to the level of technology. Using (III.1) and (III.2), Figure III.3 plots the response of annualized inflation that would be implied by our flow-based model of wage determination following a news shock using a standard value of  $\zeta = 0.8$ .

Inspection of the figure reveals that, as in the data, inflation falls on impact of the shock under both the agnostic wage process as well as the flow-based model of the wage. In the latter case, the impact response is smaller than what we find in the data, but in subsequent periods the responses are more similar. While a more complete treatment of nominal rigidities would be needed to quantitatively assess our model’s implications for inflation relative to the data, we view the results in Figure III.3 as an indication that our model is broadly consistent with the data along this dimension.

Regarding the implications of our empirical results for nominal wages, a back-of-the-envelope calculation suggests that the level of nominal wages does not actually fall in response to our shock. To see why this is, suppose that steady-state nominal wage growth is 2%

<sup>2</sup>In the context of a model with a frictional labor market, real marginal costs also depend on hiring costs. To simplify the analysis we neglect these costs, noting that previous authors have found a limited role for labor market frictions in marginal cost and thus inflation dynamics (Krause et al., 2008).

annually (or 0.5% on a quarterly basis), consistent with 2% annual steady-state inflation.<sup>3</sup> Then, to determine whether the level of nominal wages ever falls following our identified shock, we can use the observed response of wages in Figure 8 and the observed response of inflation in Figure 3 to compute the implied quarterly nominal wage inflation rate, expressed as percent deviations from the steady state. Nominal wage inflation never falls more than a quarter of a percent below its steady-state value. Because this is smaller in absolute value than the 0.5% steady-state nominal wage growth, our estimates imply that nominal wages do not actually fall in response to our identified shock.

### III.4 Suitability

Several authors have observed that, under some circumstances, VAR strategies may not be applicable to identify shocks. In particular, in some models, current and past observables may not span the space of past economic shocks, in which case static rotations of reduced-form residuals cannot correspond to the underlying economic shocks.

To address this concern, we consider our estimated baseline flow-wage model with anticipated permanent and unanticipated temporary TFP shocks as calibrated above, and extend it to include four additional shocks: (i) shocks to matching efficiency via stochastic fluctuations in  $\chi$ , (ii) shocks to labor supply via stochastic fluctuations in  $\psi$ , (iii) shocks to demand via stochastic fluctuations in  $\beta$ , and (iv) government spending shocks. We calibrate these additional shocks such that each drives a substantial portion of business cycle variation in at least one variable in our data set. Table III.4 reports the corresponding variance decomposition of the theoretical model between two and 500 quarters. Importantly, surprise and anticipated TFP shocks each account for roughly half of total variation in TFP in the model.

We then apply our exact empirical procedure to data simulated from the model, first a single extremely long sample and then 2,000 samples of the same length as our baseline data sample. This test thus accounts for functional form restrictions (i.e. 4 lags in the VAR) and finite sample bias that might appear in our estimates. Figure III.4 shows that the procedure recovers the theoretical impulse responses quite well, though not surprisingly responses are downward biased in the finite sample. For comparison, the figure also displays the average response that would be estimated on the same samples using the Kurmann and Sims (2021) approach to identifying news; these responses demonstrate a much larger impact change in TFP and a much stronger downward bias in the estimated response of employment,

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<sup>3</sup>This is actually a lower bound because productivity growth also contributes to nominal wage growth.

Table III.4: Variance decompositions of theoretical variables

Shock	TFP	Y	C	I	N	NYSE
Matching	0.00	0.09	0.09	0.08	0.13	0.07
Gov. Spending	0.00	0.03	0.13	0.02	0.04	0.04
Labor Supply	0.00	0.07	0.07	0.08	0.11	0.06
Discount Factor	0.00	0.11	0.13	0.16	0.12	0.11
Surprise TFP	0.53	0.15	0.13	0.21	0.10	0.13
News TFP	0.47	0.55	0.45	0.44	0.51	0.60

consistent with patterns we observe in the actual data.

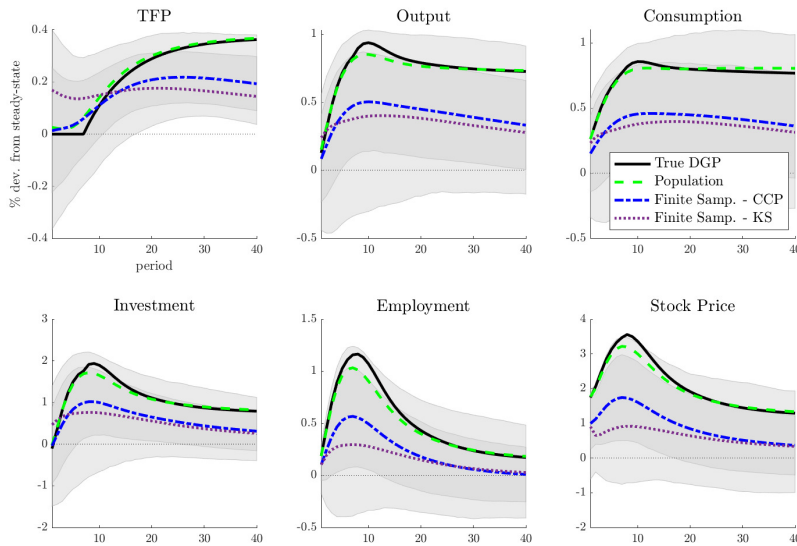


Figure III.4: Suitability exercise of empirical approach using simulated data. Dashed lines show point estimates from one 20,000 period sample. Dotted-dashed lines show the mean estimated response from 2,500 simulated samples of  $T=212$  periods using our identification strategy. Dotted lines show the corresponding object for the [Kurmann and Sims \(2021\)](#) identification strategy. Bands show the 68% and 90% interval of estimated responses from among the 2,500 model simulations.

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