Public Communication and Information Acquisition

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Abstract

This paper models the tradeoff, perceived by central banks and other public actors, between providing the public with useful information and the risk of overwhelming it with excessive communication. An information authority chooses how many signals to provide regarding an aggregate state and agents respond by choosing how many signals to observe. When agents desire coordination, the number of signals they acquire may decrease in the number released. The optimal quantity of communication is positive, but does not maximize agents' acquisition of information. In contrast to a model without information choice, the authority always prefers to provide more precise signals.

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How much should central bankers talk? Should they appoint a single speaker and provide a unified message, or allow different policymakers to discuss many topics and express dissent? Should central banks offer a detailed outlook on the economy and their objectives in setting policy? Or ought they limit themselves to very narrow statements on these topics? And, in these communications, is it better for policymakers to speak very precisely or with a degree of intentional vagueness? Central banks routinely struggle with these questions, balancing the benefits of greater transparency with the perceived risks of over-communicating. And, though communication policy varies greatly across the world's central banks, all ultimately place substantial limits on their public communication.

The priority that central banks place on crafting their communications and the preponderance of self-imposed restrictions on communication suggest that policymakers perceive tradeoffs in the choice to reveal more or less about their views on the economy. What might those tradeoffs be? The literature following Morris and Shin (2002) discusses one possibility: agents might "over-coordinate" on public information, placing more emphasis on central bank statements than is socially efficient. When public signals are imprecise, agents' over-reliance on public information may be harmful enough to warrant withholding that information altogether.

This paper provides a new account of why limits on public communication may be socially beneficial, emphasizing individuals' choices regarding the information they acquire. I propose a model in which agents' ability to coordinate their information depends on the communication policy of a benevolent information authority. The authority chooses the *scope* and *precision* of its public communications regarding an aggregate state. Scope is measured by the number of signals that the authority releases, while precision is measured by the variance of random noise contained in each signal. Agents choose the number of public signals to observe, and pay a cost for each. However, agents cannot freely coordinate on which public signals to observe. Instead, the set of signals each agent observes is randomly selected from among all public signals.

In this environment, the consequences of increasing the scope of communication depend on agents' equilibrium information choices. On the one hand, if agents acquire the additional signals, increasing scope enables them to learn more about the realization of the state and, therefore, to align their actions more closely with economic fundamentals. On the other hand, if agents do not increase acquisition one-for-one with the additional signals released, the information they observe becomes more dispersed and their actions become less coordinated. When social welfare depends on coordination, this is a cost of additional communication. Which effect predominates depends on agents' information choices and, therefore, on their strategic incentives.

This model of endogenous information choice captures the tradeoff, faced by central banks and other public actors, between providing as much information as possible and ensuring common understanding among agents. Morris and Shin (2007) argue that such considerations are important in a range of contexts, including for central banks. Similarly, Blinder (2007) expresses concern that conflicting speeches by different members of a central bank's governing body may lead to counterproductive "cacophony." And, the financial press abounds with discussions of confusion created by mixed-messages from the Federal Reserve and other central banks.[1]

[1] For example, a *Bloomberg News* article (Coy, 2012) discusses the apparent conflict between the April 2012 Federal Open Market Committee (FOMC) statement, which forecasts "exceptionally low levels for the federal funds rate at least through late 2014," and the seventeen individual FOMC member forecasts provided at the same time, only four of which show rates at zero by the end of 2014. According to the *New York Times* (Schwartz, 2013), the preponderance of disagreement during the July 2013 FOMC meeting led to such conflicting interpretations of the meeting minutes

Similar tradeoffs appear in other academic disciplines as well. Eppler and Mengis (2004) survey the idea of *information overload* spanning fields including marketing, organization science, and accounting. They define information overload as an inverted-U relationship between information provided and decision accuracy: overload occurs if information provision exceeds a threshold level, beyond which "the individual's decisions reflect a lesser utilization of the available information" (pg 328). In this model, information overload occurs rationally when individuals each receive different parts of the central bank's overall message. This diminishes the amount of information about others' actions contained in each statement made by the bank and ultimately reduces the amount of information that agents choose to acquire.

Although my model nests a canonical static coordination game, studied for example by Morris and Shin (2002) and Angeletos and Pavan (2007a), I derive a number of new results regarding the consequences of communication policy. First, I find that increases in scope have a non-monotonic impact on the amount of information acquired by agents. Revelation beyond a certain critical level causes agents to decrease the amount of information they gather. This result arises because private agents with complementarities in actions find it more desirable to obtain information that they know other agents will also obtain and act on. When the authority releases signals beyond the threshold amount, an individual agent rightfully anticipates that each signal will be observed by only a fraction of other agents and will therefore be less valuable in her own decision problem. Each agent is thus less inclined to purchase public signals, which further decreases their value, and so on. As a conse-

that, "it seemed as if [analysts] might be reading different documents." Conflicting signals from the European Central Bank have also led its recent foray into forward guidance to be criticized in the *Finacial Times* (Steen, 2013) as "unconstructive and inconsistent."

quence, too much information revelation results in less information acquisition, and lower welfare overall.

Second, despite the fact that releasing additional signals entails no direct cost for the authority, I find that the optimal level of scope is interior: the authority always releases a positive but finite amount of information. When agents and the authority value coordination equally, the logic behind this result is especially direct. Complete silence is never optimal, since the first bit of information released by the authority is acquired by all agents and therefore improves agents' forecast of the state without jeopardizing information coordination. Nor is releasing an unbounded number of signals optimal. The desire to create common understanding among agents ensures that the authority never chooses to release more information than agents willingly acquire in equilibrium.

Third, I find that the optimal degree of scope entails providing substantially less information than agents would willingly acquire if it were available. This result obtains because communication policy influences the costs of coordination faced by agents. When more signals are publicly available, agents must acquire more signals in order to ensure the same degree of coordination amongst themselves. If the authority desires coordination, then it chooses to limit this cost, even though agents would pay for and acquire extra signals if they were released by the authority.

Lastly, I find that the authority always chooses to provide the most precise signals possible. A literature characterizes cases where increases in the precision of a public signal are socially harmful (Morris and Shin, 2002; Hellwig, 2005; Angeletos and Pavan, 2007a,b). My model nests these cases, but adding the choice of scope changes the findings: the authority always prefers to adjust the number of signals it releases rather than reduce their precision. Thus, although the model rationalizes limits to the extent of a central bank's communication, it does not support a policy of "constructive ambiguity." The distinction between scope and precision offers one resolution to the incongruity of central bankers' commitment to transparency and their practice of limiting communication.

This paper proceeds as follows. Section 1 discusses related literature. Section 2 details agents' preferences and the model of information acquisition. Section 3 characterizes equilibrium in the model, and describes the consequences of communication policy for information choice. Sections 4 derives implications for optimal communication policy when agents' preferences are aligned with those of the social planner and when agents and the planner value coordination differently. Section 5 extends the model to allow agents to direct their search towards particular signals, and studies the consequences for determinacy in the model. Section 6 concludes.

1 Related Literature

This paper builds on a small but growing literature modeling the consequences of public communication for information acquisition. Most of these studies feature agents who acquire private information using a costly technology but receive public signals for free. Llosa and Venkateswaran (2012) and Colombo et al. (2012) characterize efficiency in private information acquisition in environments similar to Angeletos and Pavan (2007a). Burguet and Vives (2000) show that, when agents also freely observe the average action of other agents, more precise public information can crowd out the acquisition of private information, making aggregate actions less informative.[2] In contrast to this earlier literature, I emphasize the costs of ac-

[2] Many authors have since examined the welfare consequences of this mechanism, including Wong (2008), Colombo and Femminis (2008), Ueda (2010), and Kool et al. (2011).

quiring *public* messages and explore the implications for both the quantity and the precision of public communication. The emphasis on agents' acquisition of public information is also shared by Reis (2011), who studies the optimal timing of public announcements, and Gaballo (2013), who examines the welfare consequences of forward guidance by central banks.

The particular formulation of information acquisition used here is similar to that of Hellwig and Veldkamp (2009), but avoids the multiplicity of equilibria that they emphasize.[3] In section 5, I extend the model of information acquisition to nest both my baseline model and that of Hellwig and Veldkamp (2009). This general approach accommodates both cases with extensive multiplicity and cases with uniqueness. The model with multiple equilibria is still policy-relevant, since a policymaker who cannot predict precisely which equilibrium will arise may still wish to influence the set of possible equilibria.

This paper also builds on a literature that studies the social value of public information when information is given exogenously. My model nests that in Morris and Shin (2002), and I contrast my results with theirs in section 4. Cornand and Heinemann (2008) and Myatt and Wallace (2009) study environments where the communication authority directly controls the "publicity," or degree of common-knowledge, of its signals. In this paper, incomplete publicity of signals arises endogenously, an outcome that the authority typically seeks to avoid.

[3] Myatt and Wallace (2012) also eliminate this multiplicity by using a different formulation of the information choice.

2 Model

The model is a static *coordination game*, with endogenous information acquisition. The preference structure follows that of Angeletos and Pavan (2007a) and nests Morris and Shin (2002). Prior to choosing their actions, agents and the authority choose information in a two-stage game, in which the authority is a Stackelberg leader. In this section, I first detail agents' preferences and then describe the information game.

2.1 Preferences

The economy consists of a continuum of expected utility-maximizing agents, indexed by $i \in [0,1]$, and an information authority, denoted G. Each agent chooses an action, $p^i \in \mathbb{R}$, given their information. Agent i's preferences (exclusive of information costs, which are introduced later) are given as minus a quadratic loss function, so that

$$-u^i(p^i|\theta, p) = (1-\alpha)\left(p^i - \theta\right)^2 + \alpha(p^i - p)^2 \tag{2.1}$$

where θ represents an aggregate economic fundamental relevant to all agents, $p \equiv \int_0^1 p^i di$ gives the cross-sectional average action across agents, $\int_0^1 (p^i - p)^2 di$ denotes cross-sectional dispersion in actions, and α denotes the degree of strategic complementarity or "desire for coordination" among agents. The focus of this paper is on situations where agents place a positive value on coordinating with others in the economy, and henceforth I assume that $\alpha \in [0,1)$.[4] Under these preferences, agent

[4]Avoiding the case $\alpha = 1$ ensures uniqueness in agents' equilibrium actions, *given* their information.

i's optimal response function given her information is

$$p^{i*}(\mathcal{I}^i) = (1-\alpha)E(\theta|\mathcal{I}^i) + \alpha E(p|\mathcal{I}^i), \tag{2.2}$$

where the notation $E(x|\mathcal{I}^i)$ denotes the expectation of variable x conditional on agent i's information set, $\mathcal{I}^i$.

Social welfare in the economy is measured by

$$-u^G(\{p^i\},\theta) = (1-\alpha^\star)\int_0^1 \left(p^i-\theta\right)^2 di + \alpha^\star \int_0^1 (p^i-p)^2 di. \tag{2.3}$$

In equation (2.3), the parameter $\alpha^\star \in [0,1]$ measures the degree of complementarity from the social planner's perspective, which may be different from that perceived by agents.[5]

These preferences encompass the class of preferences considered by Angeletos and Pavan (2007a), which themselves nest the preferences of Morris and Shin (2002). Angeletos and Pavan (2007a) show that, when α equals $\alpha^\star$, agents' actions given information are optimal from a social perspective. In proposition 3, I establish that whenever α equals $\alpha^\star$ agents' individual information choices are also socially optimal. Consequently, when α equals $\alpha^\star$, I will say that the agents' and planner's preferences are "aligned."

Many models can be cast in or approximated by the preference structure described above. Hellwig (2005) and Roca (2010) derive such preferences in pricing models with imperfect competition and monetary policy, and show that coordina-

[5]The case of $\alpha \neq \alpha^\star$ is often rationalized in the related literature by assuming a "dispersion externality" in the private welfare function, $-u^i(p^i|\theta,p) = (1-\alpha)\left(p^i-\theta\right)^2 + \alpha(p^i-p)^2 + \delta\int_0^1 (p^j - p)^2 dj$, which leads the utilitarian welfare function to take the form assumed above, with $\alpha^* \equiv \frac{\alpha+\delta}{1+\delta}$. I treat $\alpha^\star$ as a independent parameter in order to simplify the subsequent analysis of optimal communication under the presence of externalities.

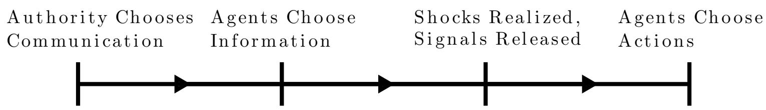

Figure 1: Timeline of the Communication Game

tion in prices is desirable at both the individual and social levels. Because of its relevance to central banking, this example motivates my initial focus on the case of aligned preferences. Other examples in which coordination is both individually and socially desirable ($\alpha \leq \alpha^{\star}$) include the model of strategic investment decisions by Angeletos and Pavan (2004) and the model of political leadership by Dewan and Myatt (2008). In other cases, such as the beauty contest models described by Morris and Shin (2002) and Allen et al. (2006) or the model of corporate board decision-making by Malenko (2011), agents desire coordination even though it brings no social benefit ($\alpha > \alpha^{\star} = 0$).

2.2 The Communication Game

Before the realization of random variables, the information authority and agents choose information in a two stage game. In the first stage, the information authority selects its communication policy, which consists of selecting the number of signals to release (scope) as well as their precision. In the second stage, each agent i chooses her individual information allocation, taking as given the information choice of other agents as well as the communication policy of the authority. Once communication policy and information allocations are made, uncertainty is realized, signals are observed, and agents choose their actions in an individually optimal manner given their information. Figure 1 summarizes the sequence events in the model.

2.2.1 Authority's Communication Policy

The aggregate state θ is normally distributed with mean zero and variance normalized to one. Its realization is known perfectly to the information authority. The authority can choose to share its information with the public by costlessly providing one or more signals, indexed by $l = 1, ..., n$, of the form

$$g_l = \theta + \eta_l. \tag{2.4}$$

The error in each signal is assumed to be gaussian white noise, i.i.d. across signals, with identical variance $\sigma_\eta^2 \geq \underline{\sigma_\eta^2}$ for all signals. Since I later show that the authority always selects the most precise (lowest σ_η^2) signals possible, I initially take this value as exogenous. The only remaining choice of the authority is scope, measured by n, the number of signals it wishes to make available for public observation. The assumption that the authority knows the state exactly is for expositional convenience only. I relax this assumption and study the consequences in appendix D.

Why is scope an interesting dimension of communication? The idea behind this formulation is that providing more information about a complex concept (e.g. "the state of economy") requires more communication on behalf of the authority or central bank. To internalize the authority's communication is costly to agents; they cannot make use of additional communication unless they expend the resources needed to process it. This gives agents' incentives an important new role in shaping the consequences of public communication. This view of communication fuses the main insights of the literature on information choice (that agents influence the information embodied in their actions) and the literature on public information (that policy-makers influence the informational environment faced by agents.)

Because of the emphasis on costly information acquisition, the scope concept maps most naturally into current debates about *how much* communication to undertake. For example, should the central bank provide the public with the details of its internal debates about policy, as is the case for the Bank of England, or with a more unified view, as the European Central Bank currently does? Should it provide forecasts of the policy rate, as a number of inflation-targeting banks do? Should it describe the details of its models and forecasting assumptions? In contrast, changes in the precision of signals are free and are necessarily "absorbed" as greater information flows to the private sector. This dimension of communication maps more naturally into the debate about "constructive ambiguity," the question of whether intentional obfuscation in a given public statement might be warranted.

2.2.2 Agent's Information Choice

Agents are Bayesian, and share a model-consistent prior on the state. In addition to any public information they choose to acquire, agents are exogenously endowed with a private signal

$$r^i = \theta + \xi^i, \tag{2.5}$$

where ξ^i is i.i.d. $N(0, \sigma_\xi^2)$ across agents.[6]

Given the authority's communication policy, each agent must choose how many of the public signals to observe, $k \leq n$, taking as given the information choices of other agents. To make these observations, agents pay a utility cost of $c(k) = \lambda k$.[7]

[6] The assumption of exogenous private information is not without loss of generality. However, as long as the cost-per-unit-precision of private information exceeds that of public information, agents will always exhaust all available public signals before considering the acquisition of additional private information. In this case, the subsequent theorems follow with very few modifications.

[7] The assumption of linear cost is for convenience only. Since the marginal benefit of information is decreasing in information acquired, convexity of $c(k)$ is sufficient but not necessary for a unique equilibrium. The qualitative results are unaffected as long as $c(k)$ is not too concave.

I assume that agent i cannot select precisely which of the n signals to observe, and instead observes a subset drawn randomly without replacement from among the signals released by the authority. This assumption is important; I discuss its relevance below and show how it may be substantially relaxed in section 5.

At the time of her actions, I assume agent i knows the realizations of k out of the n public signals and can associate each of her observations with the index of a particular signal, l.[8] With a slight abuse of notation, I will say $g_l \in \mathcal{I}^i$ whenever g_l is among the signals observed by agent i.

To choose her action, agent i must forecast both the state, θ, and the average action, p.[9] As I demonstrate in the appendix, the later requires forming expectations about both the private signals of other agents and about those public signals observed by other agents but not by agent i. These expectations can be written in terms of the conditional expectation of θ. This expectation is given by

$$E(\theta|\mathcal{I}^i) = \gamma_1 \sum_{g_l \in \mathcal{I}^i} g_l + \gamma_2 r^i, \tag{2.6}$$

where $\gamma_1 = \left(k + \sigma_\eta^2 \left(1 + \frac{1}{\sigma_\xi^2}\right)\right)^{-1}$ and $\gamma_2 = \frac{\sigma_\eta^2}{\sigma_\xi^2}\gamma_1$.

This information environment embodies two important assumptions. The first is that the communication authority cannot combine all of its signals into a single unified source of arbitrarily high precision. In the context of costly information acquisition, this is a natural assumption. Intuitively, only so much information

[8]An alternative assumption is that the agent cannot verify ex-post which of the signals she has observed. Since all signals are selected with equal probability, however, this choice is inconsequential.

[9]I prevent agents from observing a signal related to p directly, and therefore abstract from any consideration of the information aggregate actions (e.g. prices) convey to agents. The type of mechanism has been studied by Morris and Shin (2005), Amato and Shin (2006), Vives (2010), and Amador and Weill (2010), among others.

can be transmitted per unit of communication, whether that unit is a word, an image, or a speech. For the speaker to communicate more information, it must *do more communication* and agents must process it.[10] Thus, my eventual conclusion that more precision is socially beneficial implies that the communication authority should always combine their information in the most compact form possible before communicating it to the public.

The second important assumption is that agents cannot influence which of the n signals they observe. This assumption, and the assumption of Hellwig and Veldkamp (2009) that agents can *perfectly* select their preferred signals, are each special cases of a general model, in which agents search across signals with a limited ability to direct that search towards their most preferred signals. The current setup is analogous to an environment of undirected search, since agents cannot influence which of the public signals they observe. Permitting perfectly directed search, with no corresponding cost, introduces a multiplicity of equilibria into the model, for precisely the reasons described by Hellwig and Veldkamp (2009). In section 5, I extend the model to allow agents to choose the probability of observing each individual signal, nesting both of the above cases, and establish conditions on the cost of information that are sufficient to ensure that search is undirected in equilibrium.[11]

Why should search be undirected? Rational inattention provides a natural microfoundation for randomization among discrete choices, including the choice of which signals to pay attention to. For example, the entropy constraint used by Sims (2003) would imply at least some equilibrium randomization among signal choices.[12]

[10] In the language of information theory and rational inattention, information transmission requires bandwidth, which is a limited resource.

[11] One important implication of this analysis is that the authority is better off when search is undirected: when the quantity of communication is optimal, the authority has no incentive to facilitate agents directing their search.

[12] Matejka and McKay (2013) show that discrete choices taken under the rational inattention

The psychological theory of *information foraging*, advocated by Pirolli and Card (1999), also emphasizes the tradeoff individuals face between the potential value of information and the costs of seeking it out, which are distinct from the costs of information processing per-se. The information science literature also contains many theories of "information seeking," the process by which individuals select and collect information from larger menus of possible information sources. These theories emphasize the cost of searching out relevant information and the complex chains, across multiple mediums, of queries, references, and random encounters that eventually determine the information that individuals acquire.[13]

Costly coordination (via costly attention) is one plausible micro-foundation for the assumption that agents receive different messages from the authority, but the model could be extended to incorporate others as well. Agents might face individual-specific precisions or costs of acquiring various signals, reflecting the idea that individuals specialize in certain topics or find certain speakers more intelligible. Alternatively, one might view such dispersion as the result of information aggregation: real agents must acquire information about a great deal of relevant variables in order to make many different decisions and almost always gather multiple kinds of information from a single source. If each agent's choice of aggregator is driven primarily by idiosyncratic considerations, then agents again may fail to coordinate on precisely which signals they observe.[14]

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entropy constraint lead to a generalized version of the multinomial logit model, and therefore random outcomes for any positive information cost. Recently, Cheremukhin et al. (2013) have used the entropy to parameterize costly directed-search choice in a labor-matching model.

[13] For examples, see the "berry-picking" model of Bates (1989), the theories of browsing by Ellis (1989) and Chang and Rice (1993), and the model of "information encountering" by Erdelez (1997).

[14] See also Case (2012) for some discussion of mismatch between the agent's desired information and the "information units" actually available.

2.3 Definitions

In this section, I define concepts of equilibrium and efficiency, taking communication policy as given. Let $U^i(k, p^i(\mathcal{I}^i); p(G)) \equiv E[u^i] - c(k)$ be the unconditional expected utility of agent i as a function of her information allocation, k, and her action rule $p^i(\mathcal{I}^i)$, and the aggregate mapping of the state and authority's signals to aggregate actions, $p(G)$. Similarly, define $U^G(k, p^i(\mathcal{I}^i); p(G)) \equiv E[u^G] - c(k)$ to be the unconditional expected value of aggregate social welfare. Note that this notation already reflects a restriction to symmetric information allocations and action rules.

Definition 1. *Given communication policy, $\{n, \sigma_\eta^2\}$, a pure strategy symmetric equilibrium consists of an information allocation and an action rule mapping information to actions, $\{k^*, p^{i^*}(\mathcal{I}^i)\}$, such that*

1. *Each agent's choice of information and actions maximize expected utility, taking other agents' actions as given:*

$$\{k^*, p^{i^*}(\mathcal{I}^i)\} = \underset{k, p^i(\mathcal{I}^i)}{\operatorname{argmax}} \;\; U^i(k, p^i(\mathcal{I}^i); p(G)) \quad \textit{subject to} \quad k \leq n. \tag{2.7}$$

2. *The average action is given by $p(G) = \int_0^1 p^{i^*}(\mathcal{I}^i)di$.*

The restriction to pure strategy equilibria simplifies notation, and will not exclude any equilibria under assumption 1, which I present shortly and maintain throughout the paper.

I now define a benchmark that will be helpful in evaluating the efficiency of the decentralized equilibria in the model.

Definition 2. *Given a communication policy, $\{n, \sigma_\eta^2\}$, the socially optimal symmetric information-action plan consists of an i-common information allocation and an* *action rule,* $\{k^{\bullet}, p^{\bullet}(\mathcal{I}^i)\}$*, which satisfy*

$$\begin{aligned}\{k^{\bullet}, p^{i^{\bullet}}(\mathcal{I}^i)\} = &\underset{k, p^i(\mathcal{I}^i)}{\operatorname{argmax}} \quad U^G(k, p^i(\mathcal{I}^i); p(G)) \quad \text{subject to} \\ &k \leq n; \\ &p(G) = \int_0^1 p^i(\mathcal{I}^i) di.\end{aligned} \tag{2.8}$$

The definition of the planner's *information-action plan* corresponds to the efficiency criterion used in Angeletos and Pavan (2007a), in that it considers efficiency given the constraint on information. This definition extends theirs, however, by considering efficiency over a range of potential information allocations, rather than considering actions for a *fixed* information structure.

I will later establish that, when α equals $\alpha^{\star}$, a unique equilibrium satisfying definition 1 will always correspond to the socially optimal information-action plan. When α and $\alpha^{\star}$ differ, either actions, information acquisition, or both, may be inefficient.

3 Equilibrium and Efficiency

The model is solved by optimizing sequentially, first choosing actions taking information as given, and then choosing the information allocation assuming that the use of information is individually optimal.

3.1 Exogenous Information

In this section, I solve for the equilibria of the model taking the information choice of agents as given. I restrict myself to the space of linear equilibria, such that the mapping from signals to the aggregate action can be represented by a linear rule

$$p = \psi_1^* \sum_{l=1}^{n} g_l + \psi_2^* \theta. \tag{3.1}$$

In appendix A.1, I use a method of undetermined coefficients and iterate on agent's optimal action given in equation (2.2) to solve for ψ_1^* and ψ_2^*. Conditional on observing a particular public signal, individual i's response to that signal is then given by

$$\psi_1^{i\,*} = \frac{n}{k}\psi_1^* = \left(k + \left(1 - \alpha\frac{k}{n}\right)\sigma_\eta^2\left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)\right)^{-1}. \tag{3.2}$$

Compare now the expression for $\psi_1^{i\,*}$ in equation (3.2) with the optimal weight of inference on the corresponding observation, $\gamma_1 = \left(k + \sigma_\eta^2\left(1 + \frac{1}{\sigma_\xi^2}\right)\right)^{-1}$. Clearly, $\psi_1^{i\,*} = \gamma_1$ whenever agents have no strategic complementarities and they acquire all of the public signals ($\alpha = 0$ and $k = n$). Further inspection reveals that $\psi_1^{i\,*} > \gamma_1$, and $\psi_2^* < \gamma_2$, whenever agents acquires all signals but α is positive. Thus, agents "overweight" the public signal relative to the Bayesian weights.[15] This is the effect highlighted by Morris and Shin (2002): agents' desire to coordinate actions causes them to respond more strongly to public signals than they would if they sought only to align their action with the exogenous state.

3.2 Morris and Shin (2002) Result

When $k = n = 1$ and $\alpha^\star = 0$, the model nests that of Morris and Shin (2002). The fundamental result of that paper and the subsequent literature (e.g. Angeletos and Pavan, 2007a), is that social welfare may be decreasing in the precision of public

[15]Note, however, that when $\frac{k}{n}$ is sufficiently small, it may be that $\psi_1^{i\,*} < \gamma_1$. This occurs because the common prior plays a role that is identical to a common public signal. For small enough $\frac{k}{n}$, agents overweight the prior relative to all other information.

information when agents place inefficiently high weight on coordination.[16] This basic result is captured in proposition 1, which is proved in appendix A.2.

Proposition 1. *Suppose $n = k = 1$ and $\alpha^\star = 0$. Then social welfare is decreasing in $\frac{1}{\sigma_\eta^2}$ if and only if*

$$(2\alpha - 1)(1 - \alpha) > \sigma_\xi^2 \left(1 + \frac{1}{\sigma_\eta^2}\right). \tag{3.3}$$

As pointed out by Svensson (2006), the prerequisites of condition (3.3) are stringent, for it requires both that $\alpha > \frac{1}{2}$ and that private information is much more precise than public information. Explicitly including agents' common prior highlights an additional requirement: private information must also be very precise in an absolute sense. If this requirement is not met, then agents do not update their beliefs much in response to either signal, and the resulting inefficiency in actions is too small to offset the benefits of making the public signal more precise.

An important implication of proposition 1 is that an authority choosing the precision of its signal subject to an upper bound always chooses to provide the most precise signal possible or no signal at all. To see this, note that whenever the condition in (3.3) holds for a particular value $\underline{\sigma_\eta^2}$, it must also hold for values of $\sigma_\eta^2 > \underline{\sigma_\eta^2}$. In such cases the authority selects the variance of its signal to be arbitrarily large: the authority provides no information at all to the public. I follow Morris and Shin (2002) in calling this a "bang-bang" result.

3.3 Endogenous Information

In this section, I solve for the equilibrium information choice, assuming that information is used in the equilibrium manner. In appendix A.3, I derive the loss

[16] Conversely, when agents underweight coordination, increases in the precision of private information may be harmful.

function of agent i as a univariate function of k, taking the information choice and equilibrium actions of other agents as given. I now turn to deriving the equilibrium choice of k.

3.3.1 Continuous Information

For analytical simplicity, I focus on a version of the model in which the information choices of the authority and agents can each be described by continuous parameters $\hat{k}$ and $\hat{n}$, rather than the integers k and n. To transform the model into its continuous analogue, I divide each "unit" of information provided by the authority into a set of sub-units, and so on, *ad infinitum.* The details of this transformation are given in appendix A.3.1, where I show that the discrete expressions for the equilibrium coefficients and social welfare can be mapped into isomorphic expressions with $\hat{n}$ and $\hat{k}$ as continuous choice variables. Accordingly, I drop this distinction in the main text and simply treat n and k as continuous choices.

3.3.2 Equilibrium with Information Choice

Assumption 1 is maintained throughout the remainder of the paper and is necessary to ensure that agents choose to acquire a non-zero quantity of information.

Assumption 1. *Information costs are not too high:*

$$\lambda < \frac{1}{\sigma_\eta^2 \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)^2}. \tag{3.4}$$

Letting $\tau \equiv \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)$, assumption 1 reduces to $\lambda < (\hat{\sigma}_\eta \tau)^{-2}$. The assumption states that the costs of information should not be too high, relative to the precision of public information.[17] When complementarities are high, the assumption requires a relatively low cost of information. This is natural, since in this case agents are more concerned with coordination than with the value of the state *per se*. This logic is especially straightforward when agents have no private information. In this case, agents coordinate perfectly if no one acquires any information about the state.

Proposition 2, proved in appendix A.3, establishes the characteristics of equilibrium information choice in the model.

Proposition 2. *Let* $\widehat{n} \equiv \left(\frac{\sigma_\eta^2}{\lambda}\right)^{\frac{1}{2}} - (1-\alpha)\sigma_\eta^2\tau$*. Then, the equilibrium information allocation is unique and is given by*

$$k^* = \begin{cases} n & \text{if } n \leq \widehat{n} \\ \ddot{k}(n) & \text{otherwise,} \end{cases} \tag{3.5}$$

where

$$\ddot{k}(n) = \frac{\left(\frac{\sigma_\eta^2}{\lambda}\right)^{\frac{1}{2}} - \sigma_\eta^2\tau}{1 - \frac{\alpha}{n}\sigma_\eta^2\tau}. \tag{3.6}$$

The contrast between the uniqueness result here and the pervasive multiplicity in Hellwig and Veldkamp (2009) stems from the fact that, in their paper, agents may freely coordinate on the signals they wish to observe. From an agent's perspective, this creates a discontinuity in value between signals that are already observed by other agents (and therefore contain information about their actions as well as the state) and those that are not observed by others (and therefore only contain information about the state.) This discontinuity generates a range of values of information acquisition in which agents have no incentive to acquire either more or

[17] Of course, if the cost of information is so high that agents *never* acquire any information in equilibrium, then the choice of scope is irrelevant.

less information, given the information choices of other agents.[18]

Uniqueness in this case follows from the assumption that agents must randomize in selecting the signals that they observe. As a result, all signals are observed with equal probability by other agents and an agent deciding whether to observe an additional signal knows that all signals are equally informative about others' actions. Decreasing returns to additional observations arise only because agents update their beliefs less in response to new information once they are already observing a great deal of information.

3.4 Scope and Information Acquisition

How does communication policy affect equilibrium information acquisition? Corollary 1 establishes the important result that, for levels of scope beyond the critical value $\widehat{n}$, an increase in the scope of communication actually decreases the amount of information acquired by agents.

Corollary 1. *Suppose that $n > \widehat{n}$. Then information acquisition is decreasing in scope:*

$$\frac{\partial \ddot{k}(n)}{\partial n} < 0. \tag{3.7}$$

Proof of Corollary 1. To see this result, compute the derivative of $\ddot{k}(n)$:

$$\frac{\partial \ddot{k}(n)}{\partial n} = -\ddot{k}(n) \frac{\alpha \sigma_\eta^2 \tau n^{-2}}{1 - \frac{\alpha}{n} \sigma_\eta^2 \tau}. \tag{3.8}$$

[18] I am ignoring a different sort of multiplicity that occurs because agents could select to observe any set of k signals. Because I assume that signals are *a priori* identical, the set of equilibria, each focusing on a different set of k signals, are equivalent from a welfare perspective.

Since $\ddot{k}(n)$ is always positive, expression (3.8) is negative so long as the term $1-\frac{\alpha}{n}\sigma_\eta^2\tau$ is positive. Assumption 1 ensures that this term is indeed positive. □

What is the mechanism behind this result? Consider what happens as the authority increases revelation, starting from a very low level. As long as agents attend to all public signals, increasing revelation increases their learning about the state. At the threshold point $\widehat{n}$, however, agent i no longer finds it worthwhile to attend to all signals, even if she believes that other agents *do* observe all signals. When this happens, the equilibrium cannot entail agents observing all signals. However, since each agent now observes only a subset of the public signals, each signal becomes less informative about others' actions, and therefore less valuable to agent i. Each agent is now less inclined to acquire even the previous quantity of signals, and aggregate information acquisition is reduced to a level below that obtained with slightly lower scope.

One economic interpretation of this result is that over-communication on the part of the information authority, or central bank, results in harmful "cacophony" (Blinder, 2007). The central bank may, in principle, wish to communicate more information to the public, but speaking with too many voices (sending too many signals) may overload agents' interest or capacity to process that information. When this happens, extra communication is not only unhelpful, it actually reduces knowledge about the state in the private sector, which responds to the cacophony by collecting *yet less* information from the public announcements than it otherwise would.

Figure 2 plots the consequences of greater scope of communication for aggregate information acquisition. When strategic complementarities are relatively low, information acquisition reaches a maximum at $\widehat{n}$, and declines slowly thereafter.

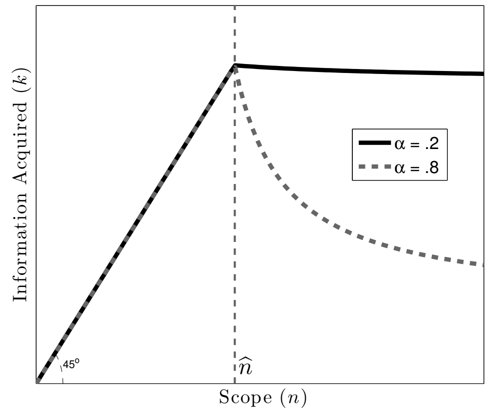

Figure 2: Information acquisition versus the scope of communication, for different degrees of strategic complementarity. Information acquisition is decreasing for scope greater than $\widehat{n}$.

However, when complementarities are relatively strong, information acquisition falls much faster.

3.5 Efficiency of Equilibrium Information

Angeletos and Pavan (2007b) establish that equivalence of α and $\alpha^\star$ is sufficient to ensure that agents' equilibrium actions are efficient, *taking their information as given.* Proposition 3 extends this result to the endogenous choice of information, for a case with a unique equilibrium. This proposition explains the decision to call α equals $\alpha^\star$ the case of "preference alignment."

Proposition 3. *Suppose $\alpha = \alpha^\star$. Then, given any communication policy, $\{n, \sigma_\eta^2\}$, the equilibrium of the model is a socially optimal information-action plan.*

Proposition 3 is a consequence of a more general theorem proved in Chahrour (2011), which states under very general conditions on the cost of information that the socially optimal action plan is an equilibrium of the model.[19] Other equilibria

[19] Let the agent's information allocation consist of a reproduction alphabet, $\hat{M}$, and a conditional

can (and often do) exist. For example, when assumption 1 does not hold, the model may have two equilibria, one in which agents acquire all of the authority's signals and one in which agents acquire no information. The different equilibria obviously have different welfare implications. The theorem establishes that one of the equilibria is efficient, but gives no guidance as to which that may be. Related results have recently been derived by Colombo et al. (2012) and Llosa and Venkateswaran (2012) in environments with unique equilibria.

4 Optimal Communication

This section studies the consequences of endogenous information acquisition for the information authority's communication policy. I first establish the basic features of optimal scope and signal precision when agent's preference are aligned with the authority's. Next, I extend the model to study the case when the authority itself is uncertain about the realization of the state. Finally, I consider the choice of communication policy when agents and the authority value coordination to different degrees.

Definition 3 formally states the information authority's problem. The information authority selects its communication policy in order to maximize social welfare.

Definition 3. *The information authority's optimal communication policy, denoted*

distribution $l(\hat{m}|\theta)$ on the message "received" by agent i, given the realization of the state. Then, for any information cost functional $c(\mathcal{I}^i)$ mapping information allocations to a real number, the optimal information-action plan is an equilibrium of the model.

by $\{n^\star, \sigma_\eta^{2\star}\}$*, maximizes the social welfare of the resulting equilibrium allocations:*

$$\begin{aligned}\{n^\star, \sigma_\eta^{2\star}\} = \underset{n,\sigma_\eta^2}{\operatorname{argmax}}\, & U^G\left(k^*, p^{i^*}(\mathcal{I}^i); p(G)\right) \quad \text{subject to} \\ & p(G) = \int_0^1 p^{i^*}(\mathcal{I}^i)di; \\ & \sigma_\eta^2 \geq \underline{\sigma_\eta^2}; \\ & k^*,\ p^{i^*}(\mathcal{I}) \textit{ are equilibrium allocations given policy } n.\end{aligned} \tag{4.1}$$

4.1 Optimal Communication with Aligned Preferences

In this section, I maintain the assumption that α equals $\alpha^\star$, and study the implications for optimal communication. I begin by focusing on this case because it ensures that agents are using their information in an efficient manner, allowing me to isolate the consequences of the coordination problem created by excessive scope. In section 4.2, I examine the robustness of these results when α and $\alpha^\star$ differ.

4.1.1 No-Waste Result

Lemma 1 establishes that, under optimal communication policy, agents must attend to all signals released by the central bank.

Lemma 1. *The optimal choice of scope induces agents to select* $k = n$*.*

The intuitive proof (detailed in appendix C.1) is as follows. Pick any communication policy n to the right of $\hat{n}$. The non-monotonicity of information acquisition implies the existence of policy n' to the left of $\hat{n}$, which achieves the same degree of information acquisition but ensures agents acquire all signals released by the authority. Under both policies, information costs are equal and agents achieve the same precision of inference about the state. However, under policy n', agents' information is more correlated and their actions are more coordinated. Therefore, policy n' always strictly dominates the higher scope policy, n.

4.1.2 Optimal Scope

Using lemma 1, I can now compute the authority's preferred scope of communication.

Proposition 4. *The optimal choice of scope is given by*

$$n^\star = \sqrt{\frac{\sigma_\eta^2(1-\alpha)}{\lambda}} - (1-\alpha)\sigma_\eta^2\tau. \tag{4.2}$$

Proposition 4 is established by (1) assuming full information acquisition on the part of agents, (2) maximizing the resulting social welfare function, and (3) checking that full information acquisition is indeed an equilibrium for the implied scope. The details of the proof are in appendix C.2.

Given the result in lemma 1, it might be tempting to guess that the authority seeks to maximize agents' information acquisition. Expression (4.2) immediately shows that this is not the case.

Corollary 2. *When $\alpha = \alpha^\star > 0$, the optimal scope is positive but entails providing fewer signals than agents would willingly acquire in equilibrium. That is,*

$$0 < n^\star < \widehat{n}. \tag{4.3}$$

Optimal communication does not saturate agents' individual demand for information. Expression (4.2) also implies that stronger complementarities increase the

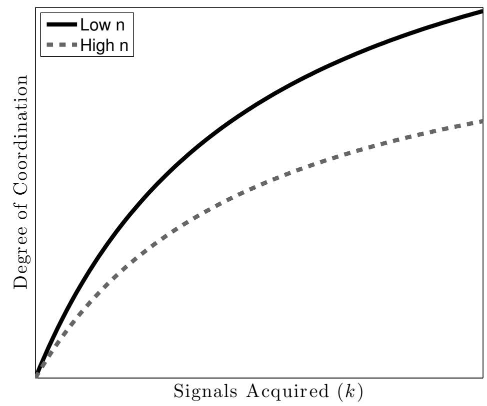

Figure 3: Information coordination (measured by the correlation of agent i's expectation with the average expectation of θ) versus the number of signal acquired, for different degrees of scope. Higher scope increases the number of signals, and therefore the information costs, required to achieve a given degree of coordination.

wedge between the number of signals agents would acquire and the amount optimally provided by the authority. Figure 3, which plots the coordination in agents' expectations as a function of the number of signals acquired for two different levels of communication, offers some intuition for this result. When agents cannot freely coordinate which signals they observe, increasing the quantity of communication increases the amount information agents must acquire to achieve the same degree of coordination in their expectations. From the perspective of private agents, increasing the scope of communication effectively increases the price of informational coordination, a negative side-effect of additional communication. Since agents value coordination, they will choose to pay this cost and acquire additional signals beyond $n^\star$, even though they would prefer that the signals were withheld entirely by the authority.

For further intuition, consider the social welfare function when agents purchase all public information:

$$-U^G\left(k^*, p^{i^*}(\mathcal{I}); p(G)\right) = (1-\alpha)\left((n\psi_1 + \psi_2 - 1)^2 + \psi_1^2 n\sigma_\eta^2\right) + \psi_2^2\sigma_\xi^2 + \lambda n. \quad (4.4)$$

Since all agents receive the same public message, the portion of the social welfare function that depends on coordination (multiplied by α) effectively disappears, leaving only a term related to alignment of actions with the fundamental (multiplied by $1-\alpha$), as well as a term related to the exogenous noise in the private signal. When α is close to one, additional signals from the authority provide relatively little benefit because the "fundamental" portion of the loss function becomes small while the cost of each signal is unchanged. Higher complementarities decrease the value of learning about the state relative to ensuring coordination in the economy, and optimal communication policy responds by decreasing the number of signals it releases.

Since complementarities drive the desire to limit communication, a natural hypothesis is that the optimal scope of communication is increasing α. In fact, this is only true when the exogenous private information is sufficiently imprecise.

Corollary 3. *Optimal scope is decreasing in α if and only if*

$$\frac{\sigma_\xi^2}{2} \geq \sqrt{(1-\alpha)\lambda\sigma_\eta^2}. \quad (4.5)$$

The logic behind the result is that increasing the degree of complementarity both decreases the importance agents place on choosing actions that are close to the state and decreases the informativeness of private information for the optimal action. When private information is of low quality, agents rely relatively little on it and the latter effect is small: agents can coordinate sufficiently well with less public information, and so the authority provides fewer signals. On the other hand, when private information is relatively precise, an increase in complementarities causes the value of private information to fall more greatly, and the authority compensates for that loss by increasing information along the dimension it controls, namely increasing public signals.

These results bear a *prima facie* resemblance to those of Morris and Shin (2002) and subsequent literature: too much communication may be harmful and this harm is driven by complementarities in agents' actions. My results differ in crucial ways, however. First, the conclusions here are not the result of any sort of "misalignment" in preferences, the driving force behind the Morris and Shin (2002) result. Instead, they stem from the nature of communication in the model, namely that communication policy can be fashioned so as to facilitate coordination among agents, and that this is desirable. The economic intuition here is quite distinct from previous literature: in this paper, limiting the extent of communication serves to facilitate desirable coordination in agents' information, while in the Morris and Shin (2002) strand of literature it serves to temper over-coordination in agents' actions.

A second important difference is that the results here rationalize partial revelation, in which the authority communicates in a manner that is neither silent nor totally revealing. In contrast, the Morris and Shin (2002) result is "bang-bang" in nature: the authority should either release its message at the highest possible precision or not at all. In this model, if the authority were constrained to choose between full revelation or complete silence, it would indeed prefer full revelation. This last statement is a consequence of proposition 3, which states that when α equals $\alpha^\star$ equilibrium allocations are efficient for any given communication policy. Since zero public information is within each agent's choice set for any level of n, that they choose k^* strictly greater than zero implies that positive scope dominates complete silence (in which case k is constrained to be zero.)

The type of limited communication called for in this model is quite different from the "limited publicity" findings of Cornand and Heinemann (2008) and Myatt and Wallace (2009), however. Here, the authority always ensures that its messages are common knowledge among agents, which is exactly what the authority in those papers seeks to avoid. This difference arises once again because those papers assume misaligned preferences, which generally lead to inefficient use of information. Under the assumptions here, actions are efficient given information. The central bank could reduce the "publicity" of it message by communicating beyond the threshold $\widehat{n}$ but it chooses optimally not to do so.

4.1.3 Optimal Precision

Proposition 5 establishes that, when preferences are aligned, the information authority always prefers to communicate as precisely as possible regardless of its choice of scope.

Proposition 5. *Given n, social welfare is improving with communication quality. That is*

$$\frac{\partial U^G}{\partial \sigma_\eta^2} < 0. \tag{4.6}$$

Proof of proposition 5. By proposition 3, it is sufficient to show that welfare is decreasing in σ_η^2, given a fixed level of k: since agents' information choice given σ_η^2 is socially efficient, any response in information acquisition can only further increase welfare. But Angeletos and Pavan (2007a) prove that, when preferences are aligned, actions given information are efficient and a decrease in precision must be socially harmful. The result follows. □

Since the precision result of Morris and Shin (2002) relies on a misalignment of preferences, proposition 5 may not be especially surprising. Yet, the model with endogenous information acquisition still calls for a limited scope of communication. This contrast highlights the distinction between the two dimensions of communication policy in the model. Even when the central bank wishes to limit the quantity of information it provides, it still would like its communications to be a precise as possible: there is no such thing as constructive ambiguity under the baseline preferences. In section 4.2, I extend this result to show that increases in the precision of public signals are welfare improving regardless of preference alignment, so long as the scope of communication is chosen optimally.

4.2 Optimal Communication with Misaligned Preferences

So far, I have emphasized the case where agents and the social planner have identical preferences. In this case, the degree to which coordination is desirable (both socially and from the agents perspective) drives the choice of scope. Yet, Morris and Shin (2002) and the subsequent literature emphasize the link between preference misalignment (or externalities) and the choice of communication policy. In this section, I examine the interaction between the information acquisition mechanism I have detailed so far, and the inefficiencies (in actions and information acquisition) that may arise when agents and the authority have different preferences. Although the basic conclusions regarding optimal communication survive, substantial nuances emerge when agents are endowed with a very precise private signal. I therefore divide this section in two, focusing first on the extreme case of no private information, before turning to the model in its full generality.

4.2.1 No Private Information

In this section, I assume that agents can only access information about the aggregate via the pronouncements of the information authority. That is, I assume that $\sigma_\xi^2 \to \infty$. In this case, the no-waste result holds and the authority's desire for coordination completely drives the choice of scope.

Lemma 1(b). *The optimal choice of scope induces agents to select $k = n$.*

The logic behind lemma 1(b) is exactly that of lemma 1, with one exception. When $k < n$, actions are generally inefficient. In this case, the authority has *greater* incentive to choose scope to ensure full acquisition. It is only in this case that equilibrium action coefficients are equal to the coefficients of inference, and the inefficiency in actions is eliminated.

Proposition 4(a). *The optimal choice of scope is given by*

$$n^\star = \sqrt{\frac{\sigma_\eta^2(1-\alpha^\star)}{\lambda}} - \sigma_\eta^2. \tag{4.7}$$

Proof of Proposition 4(a). Because there is no inefficiency in actions so long as $n \leq \widehat{n}$, agents' own complementarities have no effect on social welfare. Furthermore, when there is no private information, the threshold level $\widehat{n} = \sqrt{\frac{\sigma_\eta^2}{\lambda}} - \sigma_\eta^2$ does not depend on α. In this case, the authority can achieve its preferred outcome irrespective of α, and this outcome is achieved by $n^\star$. □

Despite the strong result in proposition 4(a), private complementarity has important implications for the consequences of sub-optimal levels of scope. Figure

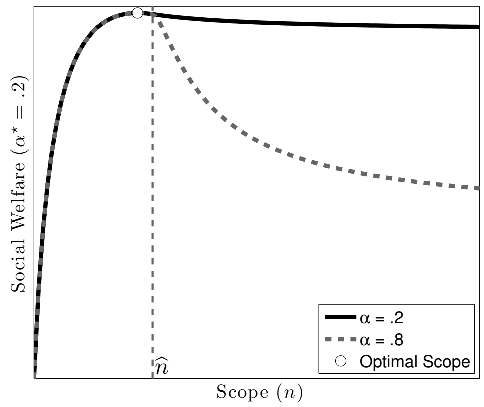

Figure 4: Social welfare as a function of the scope of communication, when agent's and the planner's preferences are misaligned. When $\alpha^\star = .2$, higher complementarities perceived by agents imply larger losses to scope beyond $\widehat{n}$.

4, compares social welfare for different degrees of private complementarities, when $\alpha^\star = .2$. When private complementarities are relatively low, "excessive" scope has smaller consequences for social welfare, since information acquisition is less affected by the extra revelation. However, when complementarities are stronger, agents react to additional signals by reducing their own acquisition of information more strongly. In this case, social welfare is more greatly harmed by the same degree of excessive communication. Importantly, however, this wedge only appears once scope exceeds the level $\widehat{n}$ at which agents cease to purchase all public information. As a result, the impact of small errors in the degree of scope depends only on $\alpha^\star$; α is only relevant in the range where information acquisition decreases with scope.

Another natural question is whether preference mis-alignment could reverse the conclusion that the authority prefers to provide the most precise signal possible. Proposition 5(a) establishes that, without private information, this result does not change.

Proposition 5(a). *The optimal communication policy entails signals of maximum* *precision:*

$$\sigma_\eta^{2*} = \underline{\sigma_\eta^2}. \tag{4.8}$$

Proof of Proposition 5(a). Since optimal scope must ensure full information acquisition, the result is immediate. Actions are optimal as long as information is common across all agents and, when actions are socially efficient, increases in the signal precision increases social welfare. □

Unlike proposition 5, proposition 5(a) only applies to the joint choice of precision and scope. Yet, the proof establishes that welfare is increasing in precision for any *fixed* level of scope that induces full acquisition. For a *fixed* level of scope beyond this threshold, however, greater precision of communication may indeed harm welfare. Surprisingly, however, this may only be true when $\alpha < \alpha^\star$. Why? This result follows from the exclusion of purely private information. In this case, when agents observe a fraction of the authority's signals, their observations become become relatively private, compared to the common prior held by all agents. Following Hellwig (2005), agents tend to overweight such signals when $\alpha < \alpha^\star$. Because they overvalue private information, agents will also tend to over-acquire these signals. In some instances with high scope, these inefficiencies can swamp the benefit of an increase in the precision of each signal. The authority, however, avoids this outcome by optimally ensuring that all agents receive the same message.

4.2.2 Private Information

Once endowed with a private signal, agents' actions may be inefficient even when they observe every signal released by the authority. This introduces a new "policy target" for the authority: it must now use its communications to influence *both* equilibrium information choice and the equilibrium use of that information. In

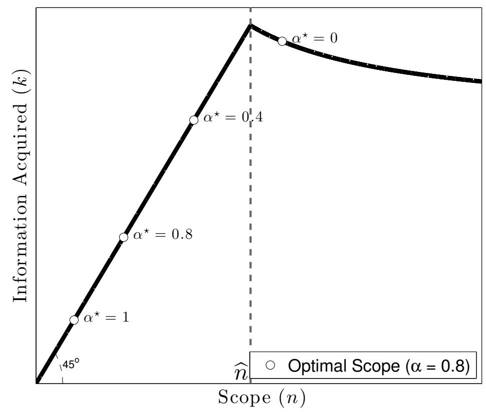

Figure 5: Optimal scope when preferences are misaligned. When preference misalignment is strong ($\alpha = 0.8$, $\alpha^\star = 0$), optimal scope may exceed the threshold level, $\widehat{n}$.

some circumstances the authority may choose scope beyond the threshold $\widehat{n}$. In these cases, the authority uses agent's inability to coordinate information to its advantage: by releasing more signals it ensures that agents experience some dispersion in their information, and therefore respond in a more muted way to each signal.

Proposition 4(b) establishes that, even though optimal communication may entail exceeding the full-acquisition threshold, it is never optimal to engage in an unbounded quantity of communication.

Proposition 4(b). *The optimal choice of scope is finite:*

$$n^\star < \infty. \tag{4.9}$$

To build some intuition for this general result, figure 5 shows optimal scope for different values $\alpha^\star$. When $\alpha \leq \alpha^\star$, a no-waste theorem applies and finite optimal scope follows immediately. On the other hand, when $\alpha > \alpha^\star$, it is indeed possible that the authority benefits from inducing some dispersion among agent's information by increasing scope beyond $\widehat{n}$. In this case, the finiteness of optimal scope follows from the presence of a common prior and the fact that the planners' preferred level of information acquisition lies above the equilibrium level. Increases in scope decrease both the level of information acquisition and the correlation of agent's signals. When agents share a prior, however, purely private information is underweighted by agents in their actions: conditional on the level of information acquisition, the authority does not desire for agents to receive purely idiosyncratic messages, as would occur with unbounded scope. Nor does the authority desire to reduce information acquisition below already inefficiently low levels. Thus, the planner necessarily settles on a finite level of scope. See appendix E.1 for the details of this proof.

Proposition 5(b). *The optimal communication policy entails signals of maximum precision:*

$$\sigma_\eta^{2\star} = \underline{\sigma_\eta^2}. \tag{4.10}$$

The argument (detailed in the appendix) is essentially as follows. Consider a small increase of precision from σ_η^2 to $\sigma_\eta^{2\prime}$. For the first case, assume the authority had optimally selected scope to ensure full acquisition when the precision of its signals was σ_η^2. Then it can always achieve higher welfare by reducing the number of signals to achieve the same precision on the combined public signal while decreasing agents' expenditure on information.

For the second case, assume the authority had selected a level of scope at which agents do not acquire all signals. If information allocations are given, then increasing the precision of the signal has the potential of generating inefficient over coordination along the lines Morris and Shin (2002). However, the authority can offset such over coordination by increasing the number of signals available to agents, thereby moving along the downward sloping portion of the information acquisition curve and causing the "publicity" of signals (i.e. the fraction of agents who see the signal) to decline. Thus, by adjusting scope appropriately, the authority can ensure that increases in precision lead to better inference without additional losses from over coordination.[20]

The result in proposition 5(b) bears some similarity to the conclusion of Cornand and Heinemann (2008) that, when public information is shared with only a fraction of the population and that fraction is selected optimally, higher precision of the public signal is always desirable. In their model, revealing a public message to a smaller fraction of the population represents an alternative, and less costly, means of inhibiting over-coordination in agents' actions. Here, revealing more information causes agents to endogenously receive more dispersed messages. In each case, the authority prefers to use an alternative dimension of communication policy, rather than reduce the precision of its message.

5 Directed Search

So far, I have assumed that agents cannot direct their search towards any particular signals among those released by the authority. In this section, I consider a version of the model in which agents choose the probability with which they observe individual signals and pay a cost for directing their search in this manner. For high-enough costs of directed search, the unique equilibrium of the generalized model is exactly that of the baseline model. When directed search is free, however, the model admits a wide-range of equilibria with different welfare implications. In this section, I argue that a policy-maker who uses robust max-min type preferences will once again prefer to limit the scope of her communications. For simplicity, I focus only on the special case in which preferences are aligned ($\alpha = \alpha^{\star}$) and agents

[20]This result holds even if the authority ignores the saved information acquisition costs.

have no exogenous private information ($\sigma_\xi^2 = \infty$).

Assume that, in addition to choosing how many signals overall to observe, agent i can also assign a sequence of weights, $\mu_1, \mu_2, ..., \mu_n$, denoting the probability that signal i is among those selected. The weights must be between zero and one and respect the restriction $\sum_{l=1}^n \mu_l = k$.[21] In order to make their observations, agents must pay a cost $c(\{\mu_l\})$. I assume that the function c is symmetric, continuously differentiable, and increasing in each of its arguments. Agent's information allocation now consists of the set $\{\mu_l\}$, however the definition of equilibrium in the extended model is otherwise identical to the definition in the baseline case.

In appendix F, I once again extend the model to a version with continuous rather than discrete choices. The solution to agent i's problem is then characterized by

$$\underset{\mu(l),k}{\text{argmax}}\ U^i \quad \text{subject to} \quad 0 \leq \mu(l) \leq 1; \int_0^n \mu(l) = k. \tag{5.1}$$

The previously cited theorem in Chahrour (2011) implies that, when α equals $\alpha^\star$, the socially optimal information-action plan is always an equilibrium of the generalized model. However, the generalized model may admit other equilibria, which entail different levels of social welfare. In section 5.2, I examine how a policy-maker can fashion a communication policy so as to maximize social welfare in the worst-case equilibrium.

5.1 A Condition for Uniqueness

I now suppose a particular functional form for $c(\mu(l))$ and show that, under a simple restriction on the cost of information, uniqueness is recovered in the model.

[21]Under the baseline assumption of undirected search, $\mu_l = \mu_j = \frac{k}{n}$ and $\sum_{l=1}^n \mu_l = n\frac{k}{n} = k$.

In particular, I assume that $c(\mu(l))$ is given by the CES aggregator,

$$c(\mu(l)) = \lambda n^{\frac{\omega-1}{\omega}} \left(\int_0^n \mu(l)^\omega \right)^{\frac{1}{\omega}}. \tag{5.2}$$

The parameter $\omega \geq 1$ measures the cost of directed search: when $\omega > 1$, agents pay more to sample the same number of signals, to the extent that they seek to observe certain signals with higher probability than they observe others. The $\frac{\omega-1}{\omega}$ exponent on n is selected to ensure that, when $\mu(l)$ is constant for all l, this specification nests the linear cost function used earlier in the paper.

Proposition 6 establishes that, for a sufficiently high cost of search, the equilibrium of the model is exactly that studied under the baseline assumptions.

Proposition 6. *Suppose that the cost of information is given by the CES aggregator in equation (5.2) and $\omega > \frac{1+\alpha}{1-\alpha}$. Then the equilibrium of the model is unique, and is characterized by*

$$\mu(l) = \frac{k^*}{n}, \forall l. \tag{5.3}$$

In this case, the policy and welfare implications are identical to the baseline model. This condition is sufficient for equilibrium uniqueness, but not necessary. For $\omega \in \left[1, \frac{1-\alpha}{1+\alpha}\right]$, the presence and extent of multiplicity will depend on the other parameters of the model. However, for these cases, the complete set of equilibria is difficult to characterize in closed form.

5.2 Robust Communication Policy

In this section, I study a version of the model in which the equilibrium information choice is not unique and study the consequences for scope. The simplest case is that in which the cost of information is linear in the number of signals sampled. This corresponds to the limiting CES case where $\omega = 1$, so that cost of information is $c(\mu(l)) = \lambda \int_0^n \mu(l)dl = \lambda k$. The set of solutions to the first order conditions (described in proposition 9 in appendix F.4) is no longer unique and includes cases where agents direct their search perfectly for all signals (choose $\mu(l) \in \{0, 1\}$ for all l), direct their search perfectly for some signals but randomly sample over others, and sample all signals with equal probability.[22]

The different equilibria have different welfare implications. If the central bank were able to choose its preferred equilibrium from among the set of all equilibria, then its choice of scope beyond $n^\star$ would be irrelevant, since it could always direct agents to ignore (its own!) extraneous signals. Such an authority could be said to be "optimistic" about the potential equilibrium outcome given its policy choice. However, if the authority is concerned that another, less desirable, equilibrium might emerge, then it may wish to take the full set of possible equilibria into account. A simple way to do this is to consider the worst case scenario - the information equilibrium with the lowest welfare - for differing degrees of scope. Such a "pessimistic" authority could then choose scope in order to place a lower-bound on the equilibrium outcome. While the source of uncertainty is different, this approach to policy making resembles the approach suggested by the *robust control* literature (e.g. Hansen and Sargent, 2001).

Figure 6 compares welfare for the best and worst case scenarios, along with the welfare generated under the baseline assumption that an agent must sample all signals with equal probability. Note that this last outcome (i.e. that of randomizing among all signals) is always an equilibrium of the model with free directed search.

[22] Again, this multiplicity is over and above the trivial sort of multiplicity that arises from interchanging the identities of the *a priori* identical signals.

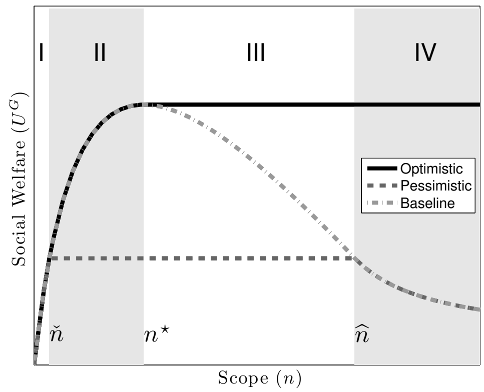

Figure 6: Social welfare for alternative equilibrium scenarios. Robust (min-max) communication policy selects n within region II or region III.

I now divide the range of scope into four regions, and describe the characteristics of each.

For the lowest levels of scope denoted by region I, all three welfare measures coincide. In these cases, the unique equilibrium is one in which agents acquire all of the signals released by the information authority. In this region, agents find it worthwhile to acquire a marginal signal, regardless of whether other agents acquire it as well.

In region II, the optimistic authority anticipates that agents will acquire all signals, while the pessimistic authority fears one of the (equally) bad outcomes in which information agents either acquire too-little information, engage in undirected search across signals, or both. Agents are therefore too-little informed about the realization of the state or too-little coordinated in their information. Once scope surpasses $\check{n}$, worst case welfare is invariant to scope, but the set of equilibria yielding this outcome becomes larger. Meanwhile, for the baseline version of the model, the randomization assumption ensures that agents acquire all information. Full acquisition is desirable throughout region II, and so the baseline model corresponds with the "best equilibrium" in this region.

In region III, the optimistic central bank remains assured that agents will focus their search on only the number of signals it knows is optimal, ignoring all others. Again, the pessimistic authority fears that agents will focus their attention too narrowly, ignoring worthwhile information, or engage in undirected search. Under the baseline assumption of sampling all signals equally, however, the agents continue to acquire all information. This outcome is suboptimal relative to the social action plan, but still better than the low-information equilibrium. Finally, in region IV, the worst-case scenario coincides with the equal-probability search assumption, as coordination decreases even as agents acquire too much information from a social perspective.

As the figure demonstrates, robust communication policy places the authority somewhere in region II or region III. In these regions, the worst case scenario entails the smallest social loss. Choosing scope in region III, which is bounded at the lower end by $n^{\star}$, ensures that the global-optimum remains among the set of equilibria. Thus, the authority can do no better than to communicate the $n^{\star}$ signals implied by the baseline model. Moreover, notice that if the authority communicates with the optimal $n^{\star}$ scope under the assumption of random search, then it has no incentive to facilitate directed search on the part of agents: doing so would only open the door to other less-desirable equilibria.

6 Conclusions and Extensions

This paper offers a new explanation for why central banks often decline to provide as much information as possible about their own views on the economy. As in the previous literature on the social value of public information, my results depend on the presence of strategic complementarities: without these strategic incentives, limiting the scope or precision of public communication never improves social welfare. Yet, in contrast to this earlier literature, my results do not rely on any sort of misalignment in preferences from the agents' and social planner's perspective. Thus, the endogenous information model used here expands the variety of contexts in which incomplete information revelation by a public authority can be optimal.

Although my model rationalizes limits on the scope of communication, it also implies that public communications should always be as precise as possible. This result helps explain why even the most ardently "transparent" central banks strictly control their public communications. My results also explain why a central bank, or other authority, might resort to *partial* revelation rather than choosing either to provide as much information as possible, or to shut down communication entirely. Avoiding such "bang-bang" outcomes increases the plausibility of the results because it accords more closely with the observation that public agencies typically release substantial but limited amounts of information.

Finally, my paper highlights the point that agents' willingness to acquire a bit of public information need not imply that releasing this information is optimal. More communication increases the cost agents must pay to coordinate information, and increasing this cost is counterproductive if coordination is socially beneficial. Conversely, when agents tend to over-coordinate on public information, releasing additional signals may be beneficial precisely because all agents *do not* acquire them. By communicating more, the central bank can prevent agents from relying too much on a narrow set of public communications.

References

Allen, F., S. Morris, and H. S. Shin (2006). Beauty Contests and Iterated Expectations in Asset Markets. *The Review of Financial Studies 19*(3), pp. 719–752.

Amador, M. and P.-O. Weill (2010). Learning From Prices: Public Communication and Welfare. *The Journal of Political Economy 118*(5), pp. 866–907.

Amato, J. and H. Shin (2006). Imperfect Common Knowledge and the Information Value of Prices. *Economic theory 27*(1), 213–241.

Angeletos, G.-M. and A. Pavan (2004). Transparency of Information and Coordination in Economies With Investment Complementarities. *The American Economic Review 94*(2), pp. 91–98.

Angeletos, G.-M. and A. Pavan (2007a). Efficient Use of Information and Social Value of Information. *Econometrica 75*(4), pp. 1103–1142.

Angeletos, G.-M. and A. Pavan (2007b). Socially Optimal Coordination: Characterization and Policy Implications. *Journal of the European Economic Association 5*(2-3), 585 – 593.

Bates, M. J. (1989). The Design of Browsing and Berrypicking Techniques For the Online Search Interface. *Online Information Review 13*(5), 407–424.

Blinder, A. S. (2007). Monetary Policy By Committee: Why and How? *European Journal of Political Economy 23*(1), 106 – 123.

Burguet, R. and X. Vives (2000). Social Learning and Costly Information Acquisition. *Economic Theory 15*(1), 185–205.

Case, D. O. (2012). *Looking For Information: a Survey of Research On Information Seeking, Needs, and Behavior* (2nd ed.). San Diego, California: Academic Press.

Chahrour, R. (2011). Central Bank Communication under Rational Inattention. Working Paper.

Chang, S.-J. and R. E. Rice (1993). Browsing: a Multidimensional Framework. *Annual Review of Information Science and Technology 28*, 231–76.

Cheremukhin, A., P. Restrepo-Echavarria, and A. Tutino (2013). A Theory of Targeted Search. Working Paper.

Colombo, L. and G. Femminis (2008). The Social Value of Public Information With Costly Information Acquisition. *Economics Letters 100*(2), 196 – 199.

Colombo, L., G. Femminis, and A. Pavan (2012, April). Information Acquisition and Welfare. Working Paper.

Cornand, C. and F. Heinemann (2008). Optimal Degree of Public Information Dissemination. *Economic Journal 118*(528), 718 – 742.

Coy, P. (2012, April 25). The Fed's Transparency is Breeding Confusion. *Bloomberg Businessweek*.

Dewan, T. and D. P. Myatt (2008). The Qualities of Leadership: Direction, Communication, and Obfuscation. *American Political Science Review 102*(03), 351–368.

Ellis, D. (1989). A Behavioural Approach to Information Retrieval System Design. *Journal of Documentation 45*(3), 171–212.

Eppler, M. J. and J. Mengis (2004). The Concept of Information Overload: a Review of Literature From Organization Science, Accounting, Marketing, MIS, and Related Disciplines. *The Information Society 20*(5), 325–344.

Erdelez, S. (1997). Information Encountering: a Conceptual Framework For Accidental Information Discovery. In *Proceedings of an international conference on Information seeking in context*, ISIC '96, London, UK, pp. 412–421. Taylor Graham Publishing.

Fisher, K. E., S. Erdelez, and L. E. McKechnie (Eds.) (2005). *Theories of Information Behavior*. Medford, New Jersey: Information Today, Inc.

Gaballo, G. (2013). Rational Inattention to News: the Perils of Forward Guidance. Banque de France Working Paper.

Hansen, L. P. and T. J. Sargent (2001). Robust Control and Model Uncertainty. *The American Economic Review 91*(2), pp. 60–66.

Hellwig, C. (2005). Heterogeneous Information and the Benefits of Transparency. Working Paper, UCLA.

Hellwig, C. and L. Veldkamp (2009). Knowing What Others Know: Coordination Motives in Information Acquisition. *Review of Economic Studies 76*(1), 223 – 251.

Kool, C., M. Middeldorp, and S. Rosenkranz (2011). Central Bank Transparency and the Crowding Out of Private Information in Financial Markets. *Journal of Money, Credit and Banking 43*(4), 765–774.

Llosa, L. G. and V. Venkateswaran (2012, March). Efficiency With Endogenous Information Choice. Working Paper.

Malenko, N. (2011). Communication and Decision Making in Corporate Boards. Working Paper.

Matejka, F. and A. McKay (2013). Rational Inattention to Discrete Choices: a New Foundation For the Multinomial Logit Model. Working Paper.

Morris, S. and H. S. Shin (2002). Social Value of Public Information. *The American Economic Review 92*(5), pp. 1521–1534.

Morris, S. and H. S. Shin (2005). Central Bank Transparency and the Signal Value of Prices. *Brookings Papers on Economic Activity 2005*(2), pp. 1–43.

Morris, S. and H. S. Shin (2007). Optimal Communication. *Journal of the European Economic Association 5*(2-3), 594–602.

Myatt, D. P. and C. Wallace (2009). On the Sources and Value of Information: Public Announcements and Macroeconomic Performance. University of Oxford, Department of Economics, Economics Series Working Papers.

Myatt, D. P. and C. Wallace (2012). Endogenous Information Acquisition in Coordination Games. *The Review of Economic Studies 79*(1), 340–374.

Pirolli, P. and S. K. Card (1999). Information Foraging. *Psycological Review 100*, 643–675.

Reis, R. (2011). When Should Policy Makers Make Announcements. Working Paper.

Roca, M. (2010). Transparency and Monetary Policy With Imperfect Common Knowledge. IMF Working Paper.

Schwartz, N. D. (2013, August 22). No Clarity From Fed On Stimulus, Upsetting Wall Street. *New York Times*.

Sims, C. A. (2003). Implications of Rational Inattention. *Journal of Monetary Economics 50*(3), 665 – 690.

Steen, M. (2013, July 15). "Loose Lips" Report Speaks Volumes About Confusion Among Central Bankers. *Financial Times*.

Svensson, L. E. O. (2006). Social Value of Public Information: Comment: Morris and Shin (2002) is Actually Pro-Transparency, Not Con. *The American Economic Review 96*(1), pp. 448–452.

Ueda, K. (2010). Central Bank Communication and Multiple Equilibria. *International Journal of Central Banking 6*(3), 145–167.

Vives, X. (2010). Endogenous Public Information and Welfare. Working Paper.

Wong, J. (2008). Information Acquisition, Dissemination, and Transparency of Monetary Policy. *Canadian Journal of Economics 41*(1), 46 – 79.

APPENDIX TO "PUBLIC COMMUNICATION AND INFORMATION ACQUISITION" FOR ONLINE PUBLICATION

A Computing Equilibrium

A.1 Equilibrium Actions

In this section, I solve for the equilibrium coefficients of the agent's action rule, taking the information structure $k \leq n$ as given. I conjecture that the aggregate action rule takes the form given in equation (3.1), derive agent i's optimal response, and compute aggregate actions given the hypothesized rule. Equilibrium is a fixed point of the resulting mapping.

Throughout, I denote with a tilde any equilibrium objects taken as given by agent i. For example, since agent i takes aggregate actions as given, the aggregate action from the perspective of agent i is assumed to be[23]

$$p = \widetilde{\psi_1} \sum_{l=1}^{n} g_l + \widetilde{\psi_2} \theta. \tag{A.1}$$

Let γ_1 and γ_2 be defined as in the text. Then, agent i's expectation of the state and average action are given respectively by

$$E(\theta|\mathcal{I}^i) = \gamma_1 \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l + \gamma_2 r^i \tag{A.2}$$

$$E(p|\mathcal{I}^i) = \widetilde{\psi_1} \sum_{l=1}^{n} E(g_l|\mathcal{I}^i) + \widetilde{\psi_2} E(\theta|\mathcal{I}^i). \tag{A.3}$$

Since I assume that agents know the identity of the signals they have observed, the

[23] Myatt and Wallace (2012) discuss the mild restrictions required to ensure the linear equilibrium is the unique equilibrium.

conditional expectation of signal l is given by

$$E\left(g_l|\mathcal{I}^i\right) = \begin{cases} g_l & \text{if } g_l \in \mathcal{I}^i \\ E(\theta|\mathcal{I}^i) & \text{if } g_l \notin \mathcal{I}^i. \end{cases} \tag{A.4}$$

After some simplification, we can compute the expectation of the aggregate action

$$E(p|\mathcal{I}^i) = \left(\widetilde{\psi_1}(1+(n-k)\gamma_1) + \widetilde{\psi_2}\gamma_1\right)\sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l + \gamma_2\left(\widetilde{\psi_1}(n-k) + \widetilde{\psi_2}\right) r^i. \tag{A.5}$$

Evaluating the agent first order condition in expression (2.2), we get agent i's choice of action as a function of her observations:

$$\begin{aligned} p^i = &\left((1-\alpha)\gamma_1 + \alpha\left(\widetilde{\psi_1}(1+(n-k)\gamma_1) + \widetilde{\psi_2}\gamma_1\right)\right)\sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l \\ &+ \gamma_2\left(1-\alpha+\alpha\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)\right) r^i. \end{aligned} \tag{A.6}$$

Rearranging the weights on the public and private signals in equation (A.6), define

$$\psi_1^i \equiv \alpha\widetilde{\psi_1} + \gamma_1\left(1-\alpha+\alpha\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)\right) \tag{A.7}$$

$$\psi_2^i \equiv \gamma_2\left(1-\alpha+\alpha\left(\widetilde{\psi_1}(n-k)+\widetilde{\psi_2}\right)\right) \tag{A.8}$$

to be the coefficients of agent i's optimal action rule, given (any) aggregate coefficients $\widetilde{\psi_1}$ and $\widetilde{\psi_2}$. In order to compute the average action, I must compute the cross-sectional average of $\mathbb{1}[g_l \in \mathcal{I}^i]g_l$.[24] By assumption, the set of signals observed is unrelated to the realizations of the signals themselves. Thus, this is just

[24] See Judd (1985); Uhlig (1996) for a discussion of the issues related to using a law of large numbers when integrating across a continuum of agents.

$E(\mathbb{1}[g_l \in \mathcal{I}^i])g_l = prob(g_l \in \mathcal{I}^i)g_l$. Because sampling is purely random, all possible combinations of signals observed are equally likely and we can immediately conclude that $prob(g_l \in \mathcal{I}^i) = \frac{k}{n}$.

Using this fact, I integrate equation (A.6) across agents to arrive at an expression for the aggregate action:

$$\begin{aligned} p =& \frac{k}{n}\left((1-\alpha)\gamma_1 + \alpha\left(\widetilde{\psi_1}(1+(n-k)\gamma_1) + \widetilde{\psi_2}\gamma_1\right)\right)\sum_{l=1}^{n} g_l \\ &+ \gamma_2\left((1-\alpha) + \alpha\left(\widetilde{\psi_1}(n-k) + \widetilde{\psi_2}\right)\right)\theta. \end{aligned} \tag{A.9}$$

Comparing equations (A.1) and (A.9), I conclude that the equilibrium coefficient is a fixed point of the recursive relationship

$$\begin{bmatrix} \psi_1' \\ \psi_2' \end{bmatrix} = (1-\alpha)\begin{bmatrix} \frac{k}{n}\gamma_1 \\ \gamma_2 \end{bmatrix} + \alpha\begin{bmatrix} \frac{k}{n}(1+(n-k)\gamma_1) & \frac{k}{n}\gamma_1 \\ (n-k)\gamma_2 & \gamma_2 \end{bmatrix}\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}. \tag{A.10}$$

Solving for the fixed point and substituting in for γ_1 and γ_2 yields the expressions

$$\psi_1^* = \left(n + \left(\frac{n}{k} - \alpha\right)\sigma_\eta^2\left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)\right)^{-1} \tag{A.11}$$

$$\psi_2^* = \left(1 + \sigma_\xi^2\left(\frac{1}{1-\alpha} + \frac{k}{1-\alpha\frac{k}{n}}\frac{1}{\sigma_\eta^2}\right)\right)^{-1}. \tag{A.12}$$

A.2 Morris and Shin (2002) Effect

Under the assumption that $n = k = 1$ and $\alpha^\star = 0$, the equilibrium action coefficients are given by

$$\psi_1^* = \left(1 + \sigma_\eta^2 \left(1 + \frac{1-\alpha}{\sigma_\xi^2}\right)\right)^{-1} \tag{A.13}$$

$$\psi_2^* = \left(1 + \frac{\sigma_\xi^2}{1-\alpha} \left(1 + \frac{1}{\sigma_\eta^2}\right)\right)^{-1}. \tag{A.14}$$

Social losses are given by

$$-U^G = (\psi_1^* + \psi_2^* - 1)^2 + (\psi_1^*)^2 \sigma_\eta^2 + (\psi_2^*)^2 \sigma_\xi^2. \tag{A.15}$$

Taking the derivative with respect to σ_η^2 yields

$$\frac{\partial U^G}{\partial \sigma_\eta^2} = -\frac{(\sigma_\xi^2)^2 \left(\sigma_\xi^2(1+\sigma_\eta^2) + (1-2\alpha)(1-\alpha)\sigma_\eta^2\right)}{\left(\sigma_\xi^2(1+\sigma_\eta^2) + (1-\alpha)\sigma_\eta^2\right)^3}, \tag{A.16}$$

which is greater than zero if any only if

$$(2\alpha - 1)(1-\alpha) > \sigma_\xi^2 \left(1 + \frac{1}{\sigma_\eta^2}\right). \tag{A.17}$$

A.3 Equilibrium Information

I now solve for agent i's choice of information, taking as given aggregate information and the equilibrium mapping of information to actions. Following the derivation above, the average action from the perspective of agent i is given by the linear rule (A.1), where

$$\widetilde{\psi_1} = \left(n + \left(\frac{n}{\widetilde{\widetilde{k}}} - \alpha \right) \sigma_\eta^2 \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2} \right) \right)^{-1} \tag{A.18}$$

$$\widetilde{\psi_2} = \left(1 + \sigma_\xi^2 \left(\frac{1}{1-\alpha} + \frac{\widetilde{k}}{1 - \alpha \frac{\widetilde{k}}{n}} \frac{1}{\sigma_\eta^2} \right) \right)^{-1} . \tag{A.19}$$

Suppose that agent i selects to observe k signals and reacts to her information optimally according to the first order condition given by (2.2). Using the weights from agent i's action rule in (A.7)-(A.8), we can compute the differences

$$p^i - \theta = \left(k\psi_1^i + \psi_2^i - 1\right)\theta + \psi_1^i \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l + \psi_2^i \xi^i \tag{A.20}$$

$$\begin{aligned} p^i - p = &\left(k\psi_1^i + \psi_2^i - n\widetilde{\psi_1} - \widetilde{\psi_2}\right)\theta + \left(\psi_1^i - \widetilde{\psi_1}\right) \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l \\ &+ \widetilde{\psi_1} \sum_{l=1}^{n} \mathbb{1}\left[g_l \notin \mathcal{I}^i\right]\eta_l + \psi_2^i \xi^i. \end{aligned} \tag{A.21}$$

From here, compute the loss function and take expectations to get

$$\begin{aligned} -U^i\left(k, p^{i^*}(\mathcal{I}); p(G)\right) = &(1-\alpha)\left(\left(k\psi_1^i + \psi_2^i - 1\right)^2 + {\psi_1^i}^2 k\sigma_\eta^2 + {\psi_2^i}^2\sigma_\xi^2\right) \\ &+ \alpha\left(\left(k\psi_1^i + \psi_2^i - n\widetilde{\psi_1} - \widetilde{\psi_2}\right)^2 + \left(\psi_1^i - \widetilde{\psi_1}\right)^2 k\sigma_\eta^2 + \left(\widetilde{\psi_1}\right)^2 (n-k)\sigma_\eta^2 + {\psi_2^i}^2\sigma_\xi^2\right) + \lambda k. \end{aligned} \tag{A.22}$$

A.3.1 Continuous Information

Fix exogenous parameters $\hat{\sigma}_\eta^2$ and $\hat{\lambda}$, which will correspond to the precision of the authority's communication and the cost of information in the continuous model. Now, consider a sequence of model indexed by parameter $\bar{n} \to \infty$, in which the public signal noise parameter is given by $\sigma_\eta^2 = \bar{n}\hat{\sigma}_\eta^2$ and the cost-per-signal is given by $\lambda = \frac{\hat{\lambda}}{\bar{n}}$. As $\bar{n}$ grows, the precision of a signal and its cost each become arbitrarily low.

The cost of information in each version of the model is invariant, in the sense that achieving a particular posterior variance on the state θ does not depend on $\bar{n}$. As an example, consider the cost of inferring the state with variance $\frac{\hat{\sigma}_\eta^2}{1+\hat{\sigma}_\eta^2}$ for a variety of $\bar{n}$. For any $\bar{n}$, doing so requires exactly $\bar{n}$ signals, since

$$E[(E(\theta|\{g_l; l = 1, ..., \bar{n}\}) - \theta)^2] = \frac{\sigma_\eta^2/\bar{n}}{1 + \sigma_\eta^2/\bar{n}} = \frac{\hat{\sigma}_\eta^2}{1 + \hat{\sigma}_\eta^2}. \tag{A.23}$$

The cost of observing $\bar{n}$ signals is always $\lambda\bar{n} = \hat{\lambda}$, establishing the invariance.[25]

Agent i's loss function can be rewritten in terms of $\hat{\sigma}_\eta^2$ and the ratios $\frac{n}{\bar{n}}$ and $\frac{k}{\bar{n}}$. The limit of this function is well-defined, so long as the limits of these ratios are also well-defined. We can now define two parameters to summarize information choices, each of which can take on a continuous (rational) value. Let $\hat{n} = \lim_{\bar{n}\to\infty} \frac{n}{\bar{n}} \in [0, \infty)$, be the information authority's choice of scope. Next, let $\hat{k} = \lim_{\bar{n}\to\infty} \frac{k}{\bar{n}}$ be agent i's information choice. Since agents can only observe those signals released by the authority, $\hat{k} \in [0, \hat{n}]$. Finally, note that cost of information can be written $c(\hat{k}) = \hat{\lambda}\hat{k}$.

As $\bar{n}$ become large, the absolute value of ψ_1^i goes to zero. Let $\hat{\psi}_1^i = \lim_{\bar{n}\to\infty} \bar{n}\psi_1^i$

[25] Expression (A.23) also establishes that, in terms of inference on the state, the information acquisition model here is identical to that of Myatt and Wallace (2012). That is, in terms of posterior variances of the state, it does not matter if I purchase $\bar{n}$ signals of variance $\sigma_\eta^2 = \bar{n}\hat{\sigma}_\eta^2$ or a single signal of variance $\hat{\sigma}_\eta^2$. The two models have very different implications for the cross-sectional correlation of information, however, which is crucial when agents interact strategically.

and $\hat{\psi}_2^i = \lim_{\bar{n}\to\infty} \psi_2^i$, so that

$$\hat{\psi}_1^i = \frac{1}{\hat{k} + \hat{\sigma}_\eta^2 + \frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}} \left((1-\alpha) + \widetilde{\hat{\psi}_1} \left(\hat{n} + \hat{\sigma}_\eta^2 + \frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2} \right) + \alpha \widetilde{\psi_2} \right) \tag{A.24}$$

$$\hat{\psi}_2^i = \frac{1}{\hat{k}\frac{\sigma_\xi^2}{\hat{\sigma}_\eta^2} + 1 + \sigma_\xi^2} \left((1-\alpha) + \alpha \widetilde{\hat{\psi}_1} (\hat{n} - \hat{k}) + \alpha \widetilde{\psi_2} \right). \tag{A.25}$$

Finally, taking the limit of expression (A.22), agent i's welfare can now be rewritten

$$-U^i \left(\hat{k}, p^{i^*}(\mathcal{I}); p(G) \right) = (1-\alpha) \left(\left(\hat{k}\hat{\psi}_1^i + \psi_2^i - 1 \right)^2 + (\hat{\psi}_1^i)^2 \hat{k} \hat{\sigma}_\eta^2 + (\hat{\psi}_2^i)^2 \sigma_\xi^2 \right)$$
$$+ \alpha \left(\left(\hat{k}\hat{\psi}_1^i - \hat{n}\widetilde{\hat{\psi}_1} \right)^2 + (\hat{\psi}_1^i - \widetilde{\hat{\psi}_1})^2 \hat{k} \hat{\sigma}_\eta^2 + \widetilde{\hat{\psi}_1}^2 (\hat{n} - \hat{k}) \sigma_\eta^2 + (\hat{\psi}_2^i - \widetilde{\psi_2})^2 + (\hat{\psi}_2^i)^2 \sigma_\xi^2 \right) + \hat{\lambda}\hat{k}. \tag{A.26}$$

Aside from the substitution of variables with ˆ's, these equations are identical to their discrete counterparts in (A.18), (A.19) and (A.22). I suppress the distinction between $\hat{n}$ and n, etc, in the paper, but maintain it in the appendix for completeness.

A.3.2 Agent Loss is Convex

Twice-differentiating (A.26) with respect to $\hat{k}$ and simplifying substantially yields

$$-\frac{\partial^2 U^i}{\partial \hat{k}^2} = 2 \frac{\hat{n}}{\hat{k}} \frac{\sigma_\xi^2 \hat{\sigma}_\eta^2}{\widetilde{\hat{\psi}_1}} \frac{\left(\hat{\sigma}_\eta^2 + \sigma_\xi^2 (\widetilde{\hat{k}} + \hat{\sigma}_\eta^2) \right)^2}{\left(\hat{\sigma}_\eta^2 + \sigma_\xi^2 (\hat{k} + \hat{\sigma}_\eta^2) \right)^3} > 0. \tag{A.27}$$

So agent i's loss (utility) is convex (concave) on $k \in [0, n]$.

A.3.3 Interior Levels of Acquisition

Agent i's problem is to find

$$\underset{\hat{k}}{\operatorname{argmax}}\ U^i \quad \text{subject to} \quad 0 \leq \hat{k} \leq \hat{n}.$$

Let λ_1 and λ_2 be the multipliers on the inequality constraints $\hat{k} \leq \hat{n}$ and $\hat{k} \geq 0$ respectively. Then the agent's first order conditions are given by

$$0 = -\frac{\partial U^i}{\partial \hat{k}} + \lambda_1 - \lambda_2 + \lambda, \tag{A.28}$$

$$\lambda_1 \geq 0; \lambda_2 \geq 0, \tag{A.29}$$

and the complementary slackness conditions. A value of $\hat{k}$ that satisfies these conditions is a unique solution to the agent's optimization problem.

Differentiating agent welfare in equation (A.26) with respect to $\hat{k}$, and imposing equilibrium conditions $\widetilde{\hat{k}} = \hat{k}$ yields the following expression:

$$-\hat{\sigma}_\eta^2 \left(\frac{\hat{n}}{\hat{k}}\hat{\psi}_1^*\right)^2 + \hat{\lambda} + \lambda_1 - \lambda_2 = 0. \tag{A.30}$$

For interior points, the extra Lagrange multipliers drop out to yield

$$\hat{\lambda} = \hat{\sigma}_\eta^2 \left(\frac{\hat{n}}{\hat{k}}\hat{\psi}_1^*\right)^2, \tag{A.31}$$

which can be solved for $\hat{k}$

$$\ddot{\hat{k}}(\hat{n}) = \frac{\left(\frac{\sigma_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2 \tau}{1 - \frac{\alpha}{\hat{n}}\hat{\sigma}_\eta^2 \tau,} \tag{A.32}$$

where $\tau = \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2}\right)$.

A.3.4 Total Information Acquisition

Full information acquisition is an equilibrium if and only if individual i's loss is (weakly) decreasing in $\hat{k}$ at $\hat{k} = \hat{n}$, when other agent's information is also full ($\widetilde{\hat{k}} = \hat{n}$.) When $\widetilde{\hat{k}} = \hat{n}$, we have that $\hat{\psi}_1^* = \left(\hat{n} + \hat{\sigma}_\eta^2 + (1-\alpha)\frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}\right)^{-1}$ and the required inequality is

$$\lambda \leq \hat{\sigma}_\eta^2 \left(\hat{n} + \hat{\sigma}_\eta^2 + (1-\alpha)\frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}\right)^{-2}. \tag{A.33}$$

Rearrange the inequality to show that full acquisition is an equilibrium whenever

$$\hat{n} \leq \left(\frac{\hat{\sigma}_\eta^2}{\lambda}\right)^{\frac{1}{2}} - \sigma_\eta^2 - (1-\alpha)\frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}. \tag{A.34}$$

A.3.5 No Information Acquisition

Conversely, no information acquisition is an equilibrium if and only if agent i's loss is (weakly) increasing in $\hat{k}$ ay $\hat{k} = 0$, when other agents' information is also nil. Taking care to avoid dividing by zero, the agent's first order condition can be rearranged when $\widetilde{\hat{k}} = 0$ to yield the condition

$$\lambda \geq \frac{\hat{\sigma}_\eta^2}{\tau^2 \left(\hat{\sigma}_\eta^2 + \hat{k}\frac{\sigma_\xi^2}{1+\sigma_\xi^2}\right)^2}. \tag{A.35}$$

Evaluating at $\hat{k} = 0$, the condition for no information acquisition to be an equilibrium is

$$\lambda \geq \frac{1}{\sigma_\eta^2 \tau^2}. \tag{A.36}$$

Some parameter constellations satisfy both conditions (A.34) and (A.36). In this case, the model has two pure strategy equilibria. However, no interior value of $\hat{k}$ can simultaneously satisfy either the full information or no information conditions equation along with condition (A.31) for an interior equilbrium. By maintaining assumption 1, uniqueness is assured and the analysis is simplified.

B Social Welfare Function

In a symmetric equilibrium for a given $\hat{n}$, $\hat{\psi}_1^i = \frac{\hat{n}}{\hat{k}}\psi_1^*$ and $\hat{\psi}_2^i = \psi_2^*$. Evaluating equilibrium actions, social welfare can be written as

$$-U^G\left(\hat{k}, p^{i^*}(\mathcal{I}); p(G)\right) = (1-\alpha^\star)\left(\left(\hat{n}\hat{\psi}_1^* + \hat{\psi}_2^* - 1\right)^2 + \left(\hat{\psi}_1^*\hat{n}\right)^2 \frac{\hat{\sigma}_\eta^2}{\hat{k}} + \left(\hat{\psi}_2^*\right)^2 \sigma_\xi^2\right)$$
$$+ \alpha^\star\left((\hat{n}\hat{\psi}_1^*)^2\left(1 - \frac{\hat{k}^*}{\hat{n}}\right)^2 \frac{\hat{\sigma}_\eta^2}{\hat{k}} + (\hat{n}\hat{\psi}_1^*)^2\left(1 - \frac{\hat{k}}{\hat{n}}\right)\frac{\hat{\sigma}_\eta^2}{\hat{n}} + \left(\hat{\psi}_2^*\right)^2\sigma_\xi^2\right) + \hat{\lambda}\hat{k}. \tag{B.1}$$

A great deal of simplification yields the following expression

$$-U^G = \hat{\sigma}_\eta^2\left(\frac{n}{k}\hat{\psi}_1^*\right)^2\left(\hat{k}\frac{1-\frac{\hat{k}}{\hat{n}}\alpha^\star}{1-\frac{\hat{k}}{\hat{n}}\alpha} + \left(1 - \frac{\hat{k}}{\hat{n}}\alpha\right)\hat{\sigma}_\eta^2\left(\frac{1}{\sigma_\xi^2} + \frac{1-\alpha^\star}{(1-\alpha)^2}\right)\right)\left(1 - \frac{\hat{k}}{\hat{n}}\alpha\right) + \hat{\lambda}\hat{k}. \tag{B.2}$$

When $\alpha = \alpha^\star$, this is just

$$-U^G = \hat{\sigma}_\eta^2\left(\frac{n}{k}\hat{\psi}_1^*\right)\left(1 - \frac{\hat{k}}{\hat{n}}\alpha\right) + \hat{\lambda}\hat{k}. \tag{B.3}$$

C Optimal Communication: Aligned Preferences

C.1 Proof of Lemma 1

Proof of Lemma 1. The form of $\hat{k}^*$ ensures that for any value $\hat{n}$ that implies $\hat{k}^*(\hat{n}) < \hat{n}$, there exists another value $\hat{n}' < \hat{n}$, such that $\hat{k}^*(\hat{n}') = \hat{n}' = \hat{k}^*(\hat{n})$. I want to show that social welfare is always higher under communication policy $\hat{n}'$. Using the simplified expression for social welfare derived in B.3, welfare under $\hat{n}'$ is greater if and only if

$$\frac{\hat{k} + (1-\alpha)\sigma_\eta^2 \tau}{\hat{k} + \left(1 - \frac{\hat{k}^*}{\hat{n}}\alpha\right)\sigma_\eta^2 \tau} \geq \frac{1-\alpha}{1 - \frac{\hat{k}^*}{\hat{n}}\alpha}. \tag{C.1}$$

But, since $\frac{\hat{k}^*}{n} < 1$ this must always be true. □

C.2 Optimal Scope

Using the result of lemma 1, I compute the optimal choice of $\hat{k}$ assuming that $\hat{k} = \hat{n}$. I then confirm that, under the implied policy, agents do choose $\hat{k}^* = \hat{n}$. If so, this represents the optimal level of transparency.

Under full acquisition, the social planner seeks to minimize loss given by

$$-U^G = (1-\alpha^\star)\left[(\hat{n}\hat{\psi}_1^* + \hat{\psi}_2^* - 1)^2 + \left(\hat{\psi}_1^*\right)^2 \hat{n}\hat{\sigma}_\eta^2\right] + \left(\hat{\psi}_2^*\right)^2 \sigma_\xi^2 + \lambda\hat{n}. \tag{C.2}$$

The first order condition is

$$-(1-\alpha^\star)\hat{\sigma}_\eta^2\left(\hat{\psi}_1^*\right)^2 + 2\left(\hat{\psi}_1^*\right)^3\left(\hat{\sigma}_\eta^2\right)^2\frac{1-\alpha}{\sigma_\xi^2}(\alpha-\alpha^\star) + \lambda = 0. \tag{C.3}$$

When preferences are aligned, this reduces to

$$(1-\alpha)\hat{\sigma}_\eta^2 \left(\hat{\psi}_1^*\right)^2 = \lambda, \tag{C.4}$$

which can be solved for $\hat{n}^\star$:

$$\hat{n}^\star = \sqrt{\frac{(1-\alpha)\hat{\sigma}_\eta^2}{\lambda}} - \hat{\sigma}_\eta^2 - (1-\alpha)\frac{\hat{\sigma}_\eta^2}{\sigma_\xi^2}. \tag{C.5}$$

Inspection shows that this is less than the threshold value $\widehat{\hat{n}}$, and so the result is established.

D Constraints on Authority's own Information

One question that arises frequently in the literature on transparency is whether the degree of the authority's knowledge about the state should impact how it communicates its message. The canonical model with a single public signal does not distinguish between limitations on the authority's knowledge of the state and limitations on its ability to clearly communicate that knowledge. In this section, I extend the model to allow for error in the authority's own knowledge of the state. The key characteristic of this type of error is that it is common across the authority's signals: every time it "speaks," the authority makes the same mistake because it misapprehends the realization of the state.

To this end, assume the authority learns about the state from a signal of the form $g = \theta + \varepsilon$. The authority, in turn, may freely release as many signals $g_l = \theta + \varepsilon + \eta_l, l = 1, ..., n$, as it wishes. The error term ε is assumed to be independent of all other shocks and normally distributed with variance σ_ε^2. All other assumptions in the model are unchanged.

D.1 Equilibrium Actions

Suppose $\sigma_\varepsilon^2 > 0$. Then, the conditional expectation of signal l is given by

$$E\left(g_l|\mathcal{I}^i\right) = \begin{cases} g_l & \text{if } g_l \in \mathcal{I}^i \\ E(\theta+\varepsilon|\mathcal{I}^i) & \text{if } g_l \notin \mathcal{I}^i. \end{cases} \tag{D.1}$$

Agent i's conditional expectation of the state and of the authority's "mistake" are now given respectively by

$$E(\theta|\mathcal{I}^i) = \gamma_1 \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l + \gamma_2 r^i \tag{D.2}$$

$$E(\varepsilon|\mathcal{I}^i) = a_1 \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l + a_2 r^i, \tag{D.3}$$

where $\gamma_1 = \frac{\sigma_\xi^2}{\chi+\sigma_\xi^2(k+\chi)}$, $\gamma_2 = \frac{\chi}{\chi+\sigma_\xi^2(k+\chi)}$, $a_1 = \frac{\sigma_\varepsilon^2\left(1+\sigma_\xi^2\right)}{\chi+\sigma_\xi^2(k+\chi)}$, $a_2 = -\frac{k\sigma_\varepsilon^2}{\chi+\sigma_\xi^2(k+\chi)}$, and $\chi = k\sigma_\varepsilon^2 + \sigma_\eta^2$.

Individual i's action is given by

$$\begin{aligned} p^i =& \left[(1-\alpha)\gamma_1 + \alpha\left(\psi_1(1+(n-k)(\gamma_1+a_1)) + \psi_2\gamma_1\right)\right] \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l \\ &+ \left[(1-\alpha)\gamma_2 + \alpha\left(\psi_1(n-k)(\gamma_2+a_2) + \psi_2\gamma_2\right)\right] r^i. \end{aligned} \tag{D.4}$$

Once again computing the expectation of the aggregate action gives

$$
\begin{aligned}
E(p|\mathcal{I}^i) = (\psi_1(1+(n-k)(\gamma_1+a_1)) + \psi_2\gamma_1) \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l \\
+ (\psi_1(n-k)(\gamma_2+a_2) + \psi_2\gamma_2)\, r^i.
\end{aligned}
\tag{D.5}
$$

Finding the fixed point as before yields the equilibrium coefficients,

$$
\psi_1^* = \left(n\Gamma + \left(\frac{n}{k} - \alpha\right) \sigma_\eta^2 \left(\frac{1}{1-\alpha} + \frac{1}{\sigma_\xi^2} \right) \right)^{-1} \tag{D.6}
$$

$$
\psi_2^* = \left(\Gamma + \sigma_\xi^2 \left(\frac{1}{1-\alpha} + \frac{k}{(1-\alpha)k\sigma_\varepsilon^2 + \left(1 - \alpha\frac{k}{n}\right)\frac{1}{\sigma_\eta^2}} \right) \right)^{-1}, \tag{D.7}
$$

where $\Gamma \equiv 1 + \sigma_\varepsilon^2 + (1-\alpha)\frac{\sigma_\varepsilon^2}{\sigma_\xi^2}$.

D.2 Equilibrium Information

The agent's problem is exactly as in A.3.3 and the first order conditions are identical (up to the relevant definition of $\hat{\psi}_1^*$.)

$$
-\hat{\sigma}_\eta^2 \left(\frac{n}{k}\hat{\psi}_1^*\right)^2 + \lambda + \lambda_1 - \lambda_2 = 0. \tag{D.8}
$$

The derivations of equilibrium information follow exactly as before. One potentially surprising result is that the condition for zero information to be an equilibrium does not change. That is, there is no need to restate assumption 1 as long $\sigma_\varepsilon^2 < \infty$.

Proposition 2(c) describes the equilibrium information choice of agents

Proposition 2(c). *Suppose that assumption 1 holds and that σ_ε^2 is finite. Then the equilibrium information allocation is unique and is given by*

$$k^* = \begin{cases} n & \textit{if } n \leq \left(\sqrt{\frac{\sigma_\eta^2}{\lambda}} - (1-\alpha)\sigma_\eta^2\tau\right)\frac{1}{\Gamma} \\ \ddot{k}(n) & \textit{otherwise} \end{cases} \tag{D.9}$$

where

$$\ddot{k}(n) = \frac{\sqrt{\frac{\sigma_\eta^2}{\lambda}} - \sigma_\eta^2\tau}{\Gamma - \frac{\alpha}{n}\sigma_\eta^2\tau}. \tag{D.10}$$

Since $\Gamma > 1$, the threshold at which agents cease to observe all signals released by the authority necessarily shrinks. Furthermore, to the right of the threshold, the derivative

$$\frac{\partial\ddot{k}(n)}{\partial\sigma_\varepsilon^2} = -\ddot{k}(n)\frac{\sigma_\varepsilon^2\left(1+\frac{1-\alpha}{\sigma_\xi^2}\right)}{\Gamma - \frac{\alpha}{n}\sigma_\eta^2\tau} < 0. \tag{D.11}$$

Thus, agent's acquisition of public signals is always less when the authority knows less about the state.

D.3 Choice of Scope

Social welfare under the assumption $\hat{k} = \hat{n}$ is now written

$$U^G = (1-\alpha)\left[\left(\hat{n}\hat{\psi}_1^* + \hat{\psi}_2^* - 1\right)^2 + \left(\hat{\psi}_1^*\right)^2\hat{n}\hat{\sigma}_\eta^2 + \left(\hat{n}\hat{\psi}_1^*\right)^2\sigma_\varepsilon^2\right] + \left(\hat{\psi}_2^*\right)^2\sigma_\xi^2 + \lambda\hat{n}. \tag{D.12}$$

Taking the first order condition and solving for $\hat{n}^\star$ yields

$$\hat{n}^\star = \left(\sqrt{\frac{\hat{\sigma}_\eta^2(1-\alpha)}{\hat{\lambda}}} - (1-\alpha)\hat{\sigma}_\eta^2\tau\right)\frac{1}{\Gamma}. \tag{D.13}$$

Proposition 4(c) summarizes optimal scope in this case.

Proposition 4(c). *Suppose that assumption 1 holds and that σ_ε^2 is finite. Then the* *optimal choice of scope is given by*

$$n^{\star} = \left(\sqrt{\frac{\sigma_\eta^2(1-\alpha)}{\lambda}} - (1-\alpha)\sigma_\eta^2\tau \right) \frac{1}{\Gamma}. \tag{D.14}$$

The proposition shows that optimal scope with $\sigma_\varepsilon^2 > 0$ is just a rescaling of the optimal level when the authority's information is perfect. Proposition 4(a) confirms an intuitive result: an authority which knows less about the state should provide fewer public signals about it.

Since errors in the authority's knowledge of the state are common across all public signals, they bear on the informativeness of the public signals with regard to the fundamental, but are not directly related to agents' coordination problem. For this reason, it is natural to suppose that increasing the magnitude of such errors is always harmful to social welfare. Proposition 7 establishes this result for the case of aligned preferences.

Proposition 7. *Given n, equilibrium social welfare is increasing in the precision of the authority's own information. That is,*

$$\frac{\partial U^G}{\partial \sigma_\varepsilon^2} < 0. \tag{D.15}$$

Proof. Again, by proposition 3, it is sufficient to show this holds for a given choice of k. The proposition then follows directly from the fact that actions given information are efficient when preferences are aligned. □

In summary, the addition of uncertainty in the authority's own apprehension of the state confirms some natural conjectures, but it does not affect the basic mechanism of the model. The results on information acquisition and optimal scope follow with minimal modification.

E Optimal Communication: Misaligned Preferences

E.1 Private Information

Proof of Proposition 4(b). Suppose $\hat{\sigma}_\eta^2$ is given. Then, when $\alpha \leq \alpha^\star$, the authority always prefers that agents acquire full information. To see this compare social welfare for a given $\hat{k}$, for the case that $\hat{k} < \hat{n}$ and the case that $\hat{k} = \hat{n}$. In many cases, full acquisition may also be optimal when $\alpha > \alpha^\star$ as well. In these cases, it follows directly that $\hat{n}^\star$ is finite.

Now suppose $\alpha > \alpha^\star$ and the the authority finds it optimal to select $\hat{n}^o > \widehat{\hat{n}}$. To find $\hat{n}^o$ in this case, plug in agents' information acquisition $\ddot{k}(\hat{n})$, take the first order condition with respect to $\hat{n}$, and solve. This first order condition has a unique (real) solution, $\hat{n}^o$, and it is finite. To show that this is indeed an optimum, however, requires slightly more work. Taking second order condition and evaluating at $\hat{n}^o$ shows that the loss function is locally convex, and since there is only one critical point, that this is indeed the global optimum.

The general expression for $\hat{n}^o$ is unwieldy. However, when $\alpha^\star = 0$ it simplifies substantially to

$$\hat{n}^o = (1-\alpha)^2 \left(\sqrt{\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}} \left(\frac{1}{(1-\alpha)^2} + \frac{1}{\sigma_\xi^2} \right) - \hat{\sigma}_\eta^2 \tau^2 \right). \tag{E.1}$$

□

Proof of Proposition 5(b). It suffices to show that welfare is improving in $\frac{1}{\hat{\sigma}_\eta^2}$, given optimal scope, for any level $\hat{\sigma}_\eta^2$. If optimal scope calls for full acquisition, the authority can always decrease scope to maintain the same total precision and benefit from lower information costs. I now demonstrate that when optimal scope lies to the right of $\widehat{\hat{n}}$, welfare is again improving if scope is adjusted optimally. Substitute in $\ddot{k}(\hat{n})$, and then $\hat{n}^o$, into the social welfare function given in (B.2). The derivative with respect to $\hat{\sigma}_\eta^2$ is greater than zero by assumption 1, and the result is established. □

F Directed Search

In this section, I solve a version of the model with a more general type of information choice, in which agents may choose the probability with which they observe particular signals, but at a cost. The logic parallels that of section A.1, despite some added complications.

Assume now that agents assign relative weights $w_1, w_2, ..., w_n$ to each signal released by the information authority, so that the probability of drawing the j'th signal as the first signal drawn is $\frac{w_j}{\sum_{l=1}^{n} w_l}$. I again assume that signals are drawn sequentially, without replacement. Because signals are drawn without replacement, the probability that g_j is drawn on the second draw depends on which signal was drawn in the first round, and so on. The distribution characterizing this search process is know as the generalized *Wallenius noncentral hypergeometric distribution.*

Let $\mu_l = P(\mathcal{G}_l^i = 1; ||\mathcal{G}^i|| = k)$ be the probability that signal l is drawn among a sample of k signals. Unfortunately, for $k > 1$ there is no requirement that μ_l is proportional to w_l and, in fact, there is no closed-form solution for μ_l as a function of the w_l's. Chen et al. (1994) show, however, that a set of w_l's can be mapped unquietly into a set of μ_l's and the two respect a natural ordering relation

$$w_l > w_j \iff \mu_l > \mu_j. \tag{F.1}$$

To simplify the analysis, and because the agents care directly about μ_l, I proceed as if these are the fundamental choice of the agents, although they could always be mapped back into a set of weights used for the sampling process. The μ_l's also have the important property that $\sum_{l=1}^{n} \mu_l = k$.

F.1 Equilibrium Actions

The equilibrium pricing rule must reflect the fact that some signals may, in general, be observed by more agents than others. Therefore I guess the following form for a linear equilibrium

$$p = \sum_{l=1}^{n} \tilde{\psi}_l g_l. \tag{F.2}$$

Under the baseline information assumptions, we have that

$$E(\theta|\mathcal{I}^i) = \frac{1}{k + \sigma_\eta^2} \sum_{l=1}^{n} \mathbb{1}\left[g_l \in \mathcal{I}^i\right] g_l \tag{F.3}$$

$$E(g_l|\mathcal{I}^i) = \begin{cases} g_l & \text{if } g_l \in \mathcal{I}^i \\ E(\theta|\mathcal{I}^i) & \text{if } g_l \notin \mathcal{I}^i. \end{cases} \tag{F.4}$$

Optimal action on the part of agent i implies

$$\begin{aligned}
p^i &= \frac{(1-\alpha)}{k+\sigma_\eta^2}\sum_{l=1}^{n}\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l + \alpha\sum_{l=1}^{n}\tilde{\psi}_l\left[\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l + \mathbb{1}\left[g_l \notin \mathcal{I}^i\right]\frac{1}{k+\sigma_\eta^2}\sum_{j=1}^{n}\mathbb{1}\left[g_j \in \mathcal{I}^i\right]g_j\right] \\
&= \sum_{l=1}^{n}\left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\tilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2}\sum_{j=1}^{n}\tilde{\psi}_j\mathbb{1}\left[g_j \notin \mathcal{I}^i\right]\right]\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l \\
&\equiv \sum_{l=1}^{n}\hat{\psi}_l^i\mathbb{1}\left[g_l \in \mathcal{I}^i\right]g_l. \qquad \text{(F.5)}
\end{aligned}$$

where $\hat{\psi}_l^i$ is agent i's optimal response to signal l conditional on $g_l \in \mathcal{I}^i$.

At this point, a new complication arises in that $\hat{\psi}_l^i$ is a random variable, both cross sectionally and from the perspective of agent i. This randomness is problematic because $\hat{\psi}_l^i$ and $\mathbb{1}[g_l \in \mathcal{I}^i]$ are not independent and no closed form expression exists for their covariance. This complicates the step of integrating across agents in order to determine the aggregate action rule. The key observation required to circumvent this difficulty is that, as $\bar{n}$ grows larger, $\hat{\psi}_l^i$ becomes essentially deterministic. This allows for both straightforward aggregation across agents and simple computation of expected values.

To make this claim more precise, consider once again a sequence of models indexed by $\bar{n}$, in which $\lim_{\bar{n}\to\infty}\frac{n}{\bar{n}} = \hat{n}$ and $\lim_{\bar{n}\to\infty}\frac{k}{\bar{n}} = \hat{k}$, and $\sigma_\eta^2 = \bar{n}\hat{\sigma}_\eta^2$. Define the set of random variables $x_j = n\widetilde{\psi}_{\widetilde{l}_j}\mathbb{1}\left[g_{\widetilde{l}_j} \notin \mathcal{I}^i\right]$, where the indexes $\widetilde{l}_j, j = 1, 2, ..., n$, are generated by randomly drawing an index l, without replacement, from among the n public signals. Define

$$b_{\bar{n}} \equiv E[x_1] = \frac{1}{n}\sum_{l=1}^{n}n\widetilde{\psi}_l(1-\mu_l) \qquad \text{(F.6)}$$

$$\delta_{\bar{n}} \equiv \frac{1}{n}\sum_{j=1}^{n}n\widetilde{\psi}_l\mathbb{1}\left[g_j \notin \mathcal{I}^i\right] - b_{\bar{n}} = \frac{1}{n}\sum_{j=1}^{n}x_j - b_{\bar{n}}. \qquad \text{(F.7)}$$

Then, equation (F.5) can then be rewritten after some manipulation as

$$p^i = \sum_{l=1}^{n} \left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\widetilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2} \left(b_{\bar{n}} + \delta_{\bar{n}} \right) \right] \mathbb{1} \left[g_l \in \mathcal{I}^i \right] g_l. \tag{F.8}$$

Integrate across agents to get

$$p = \sum_{l=1}^{n} \left(\frac{\alpha}{k+\sigma_\eta^2} \Delta_{\bar{n},l} + \mu_l \left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\widetilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2} b_{\bar{n}} \right] \right) g_l, \tag{F.9}$$

where $\Delta_{\bar{n},l} \equiv E\left[\delta_{\bar{n}} \mathbb{1}[g_l \in \mathcal{I}^i]\right]$. The equilibrium coefficients are then given by the fixed point of the expression

$$\widetilde{\psi}_l = \frac{\alpha}{k+\sigma_\eta^2} \Delta_{\bar{n},l} + \mu_l \left[\frac{1-\alpha}{k+\sigma_\eta^2} + \alpha\widetilde{\psi}_l + \frac{\alpha}{k+\sigma_\eta^2} b_{\bar{n}} \right]. \tag{F.10}$$

Now, solving for $\widetilde{\psi}_l$ yields

$$\widetilde{\psi}_l = \frac{\mu_l}{1-\alpha\mu_l} \frac{1}{k+\sigma_\eta^2} \left(1-\alpha+\alpha b_{\bar{n}} \right) + \frac{\alpha}{(1-\alpha\mu_l)(k+\sigma_\eta^2)} \Delta_{\bar{n},l}. \tag{F.11}$$

Now, using the fact that $-E(|\delta_{\bar{n}}|) \leq \Delta_{\bar{n},l} \leq E(|\delta_{\bar{n}}|)$, we have the inequality

$$\widetilde{\psi}_l \leq \frac{\mu_l}{1-\alpha\mu_l} \frac{1}{k+\sigma_\eta^2} \left(1-\alpha+\alpha b_{\bar{n}} \right) + \frac{\alpha}{(1-\alpha\mu_l)(k+\sigma_\eta^2)} E(|\delta_{\bar{n}}|) \tag{F.12}$$

and a corresponding lower bound on $\widetilde{\psi}_l$. Substituting recursively and simplifying yields the following bounds on $\widetilde{\psi}_l$

$$\frac{\mu_l}{1-\alpha\mu_l}\rho_1 - E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\rho_{2,l} \leq \widetilde{\psi}_l \leq \frac{\mu_l}{1-\alpha\mu_l}\rho_1 + E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\rho_{2,l}, \tag{F.13}$$

where

$$\rho_1 = \frac{1-\alpha}{k+\sigma_\eta^2-\alpha q}$$
$$\rho_{2,k} = \left(\frac{1}{k+\sigma_\eta^2} + \frac{\alpha\mu_l\bar{q}}{(k+\sigma_\eta^2)(k+\sigma_\eta^2-\alpha q)}\right)$$
$$q = \sum_{l=1}^{n}\frac{(1-\mu_l)\mu_l}{1-\alpha\mu_l}$$
$$\bar{q} = \sum_{l=1}^{n}\frac{(1-\mu_l)}{1-\alpha\mu_l}.$$

Now, multiply the inequality by $\bar{n}$, to get

$$\frac{\mu_l}{1-\alpha\mu_l}\bar{n}\rho_1 - E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\bar{n}\rho_{2,k} \leq \bar{n}\widetilde{\psi}_l \leq \frac{\mu_l}{1-\alpha\mu_l}\bar{n}\rho_1 + E(|\delta_{\bar{n}}|)\frac{\alpha}{(1-\alpha\mu_l)}\bar{n}\rho_{2,k}. \tag{F.14}$$

A law of large numbers applies to $\frac{1}{n}\sum_{j=1}^{n}x_j$, implying that $\lim_{\bar{n}\to\infty}E(|\delta_{\bar{n}}|)=0$.[26] Let

$$Q \equiv \lim_{\bar{n}\to\infty}\frac{1}{\bar{n}}\sum_{l=1}^{n}\frac{(1-\mu_l)\mu_l}{1-\alpha\mu_l}. \tag{F.15}$$

This is clearly finite, since each term in the summand is positive and bounded by a finite constant, while the total is divided by n. For the same reasons, $\bar{Q} \equiv \lim_{\bar{n}\to\infty}\frac{1}{\bar{n}}\sum_{l=1}^{n}\frac{1-\mu_l}{1-\alpha\mu_l}$ is also finite. Therefore, $\bar{n}\rho_1$ and $\bar{n}\rho_{2,l}$ each converge to a finite values and we can conclude that

$$\lim_{\bar{n}\to\infty}\bar{n}\widetilde{\psi}_l = \frac{\mu_l}{1-\alpha\mu_l}\rho \qquad \equiv \widetilde{\varphi}(l) \tag{F.16}$$
$$\lim_{\bar{n}\to\infty}\bar{n}\hat{\psi}_l = \frac{1-\alpha}{\hat{k}+\hat{\sigma}_\eta^2} + \alpha\widetilde{\varphi}_l + \frac{\alpha}{\hat{k}+\hat{\sigma}_\eta^2}\sum_{k=0}^{\infty}(1-\mu_l)\widetilde{\varphi}_l \qquad \equiv \varphi(l), \tag{F.17}$$

[26] This follows from the construction of x_j as a sequence of *exchangeable* random variables. See McCall (1991) for a detailed discussion and additional references on the topic of exchangeability.

where $\rho = \frac{1-\alpha}{k+\hat{\sigma}_\eta^2-\alpha Q}$.

F.2 Agents' Information Choice

I now follow a similar strategy to compute the loss of agent i, taking aggregate actions as given. To begin, compute the deviations

$$p^i - \theta = \left(\sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - 1\right)\theta + \sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l \tag{F.18}$$

$$p^i - p = \left(\sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - \widetilde{\psi}_l\right)\theta + \sum_{l=1}^{n} \left(\hat{\psi}_l - \widetilde{\psi}_l\right)\mathbb{1}\left[g_l \in \mathcal{I}^i\right]\eta_l + \sum_{l=1}^{n} \widetilde{\psi}_l \mathbb{1}\left[g_l \notin \mathcal{I}^i\right]\eta_l \tag{F.19}$$

of the discrete model. We are interested in computing $E\left[(p^i-\theta)^2\right]$ and $E\left[(p^i-p)^2\right]$.

First, consider the "fundamental deviation" given by

$$\begin{aligned}
E\left[(p^i-\theta)^2\right] &= E\left(\sum_{l=1}^{n} \hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - 1\right)^2 + \sum_{l=1}^{n} E(\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right])^2 \sigma_\eta^2 \\
&= E\left(\frac{1}{\bar{n}}\sum_{l=1}^{n} \bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] - 1\right)^2 + \frac{1}{\bar{n}}\sum_{l=1}^{n} E(\bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right])^2 \frac{\sigma_\eta^2}{\bar{n}} \\
&= E\left[\left(\frac{1}{\bar{n}}\sum_{l=1}^{n} \bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right]\right)^2 - 2\frac{1}{\bar{n}}\sum_{l=1}^{n} \bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right] + 1\right] \\
&\quad + \frac{1}{\bar{n}}\sum_{l=1}^{n} E(\bar{n}\hat{\psi}_l \mathbb{1}\left[g_l \in \mathcal{I}^i\right])^2 \frac{\sigma_\eta^2}{\bar{n}}.
\end{aligned} \tag{F.20}$$

Taking the limit $\bar{n} \to \infty$ and rewriting the infinite sum as an integral over the domain $l \in [0, \hat{n}]$, expression (F.20) now simplifies considerably to

$$E\left[(p^i-\theta)^2\right] = \left(\int_o^{\hat{n}} \mu(l)\varphi(l)dl - 1\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\varphi(l)^2 dl. \tag{F.21}$$

The limiting "coordination loss" term can be derived in the same manner:

$$E\left[(p^i - p)^2\right] = \left(\int_0^{\hat{n}} \mu(l)\left(\varphi(l) - \widetilde{\varphi}(l)\right) dl\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\left(\varphi(l) - \widetilde{\varphi}(l)\right)^2 dl \\ + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} (1 - \mu(l))\widetilde{\varphi}(l)^2 dl. \tag{F.22}$$

Finally, write the cost of information as the functional mapping $\mu(l)$ to the cost $c(\mu(l))$. Now, combining all terms yields agent i's welfare function

$$\begin{aligned} -U^i =&(1-\alpha)\left[\left(\int_o^{\hat{n}} \mu(l)\varphi(l)dl - 1\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\varphi(l)^2 dl\right] \\ &+ \alpha\left[\left(\int_0^{\hat{n}} \left(\mu(l)\varphi(l) - \widetilde{\varphi}(l)\right) dl\right)^2 + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(l)\left(\varphi(l) - \widetilde{\varphi}(l)\right)^2 dl \right. \\ &\left. + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} (1 - \mu(l))\widetilde{\varphi}(l)^2 dl\right] + c(\mu(l)). \end{aligned} \tag{F.23}$$

The solution to agent i's problem is characterized by

$$\underset{\mu(l),\hat{k}}{\text{argmax}}\; U^i \qquad \text{subject to} \qquad \mu(l) \leq 1; \mu(l) \geq 0; \int_0^{\hat{n}} \mu(l)dl \leq \hat{k}.$$

Let $\lambda_1(l), \lambda_2(l), \lambda_3$ be Lagrange multipliers on the three constraints respectively.

F.2.1 Equilibrium Characterization

Taking derivatives with respect to $\mu(l)$ and $\hat{k}$ yields

$$
\begin{aligned}
0 = 2(1-\alpha) & \left[(\phi - 1) \left(\varphi(l) - \frac{\alpha \hat{k}}{\hat{k} + \hat{\sigma}_\eta^2} \widetilde{\varphi}(l) \right) + \hat{\sigma}_\eta^2 \left(\frac{\varphi^2(l)}{2} - \frac{\alpha \widetilde{\varphi}(l)}{\hat{k} + \hat{\sigma}_\eta^2} \int_0^{\hat{n}} \mu(j)\varphi(j)dj \right) \right] \\
& + 2\alpha \left[\left(\phi - \int_0^{\hat{n}} \widetilde{\varphi}(j)dj \right) \left(\varphi(l) - \frac{\alpha \hat{k}}{\hat{k} + \hat{\sigma}_\eta^2} \widetilde{\varphi}(l) \right) + \hat{\sigma}_\eta^2 \left(\frac{\varphi(l)^2}{2} - \varphi(l)\widetilde{\varphi}(l) \right. \right. \\
& - \left. \left. \widetilde{\varphi}(l) \frac{\alpha}{\hat{k} + \hat{\sigma}_\eta^2} \int_0^{\hat{n}} \mu(j)(\varphi(j) - \widetilde{\varphi}(j))dj \right) \right] + \lambda_1(l) - \lambda_2(l) + \lambda_3 + c_l(\mu(l))
\end{aligned} \tag{F.24}
$$

$$
\begin{aligned}
-\lambda_3 = 2(1-\alpha) & \frac{\partial \varphi}{\partial \hat{k}} \left[(\phi - 1) \int_0^{\hat{n}} \mu(j)dj + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(j)\varphi(j)dj \right] \\
& + 2\alpha \frac{\partial \varphi}{\partial \hat{k}} \left[\left(\phi - \int_0^{\hat{n}} \widetilde{\varphi}(j)dj \right) \int_0^{\hat{n}} \mu(j)dj + \hat{\sigma}_\eta^2 \int_0^{\hat{n}} \mu(j) \left(\varphi(j) - \widetilde{\varphi}(j) \right) dj \right],
\end{aligned} \tag{F.25}
$$

where $\phi = \int_0^{\hat{n}} \mu(l)\varphi(l)dl$ and

$$
\frac{\partial \varphi}{\partial \hat{k}} \equiv - \left(\frac{1}{\hat{k} + \hat{\sigma}_\eta^2} \right)^2 \left((1-\alpha) + \alpha \int_0^{\hat{n}} (1 - \mu(j))\widetilde{\varphi}(j)dj \right) = \frac{\partial \varphi(l)}{\partial \hat{k}} \tag{F.26}
$$

is constant across l.

Substituting the equilibrium relationships

$$
\hat{k} = \int_0^{\hat{n}} \mu(j)dj \tag{F.27}
$$

$$
\widetilde{\varphi}(l) = \mu(l)\varphi \tag{F.28}
$$

$$
\varphi = \left(\frac{1}{1 - \alpha\mu(l)} \right) \rho \tag{F.29}
$$

into equation (F.24) and simplifying substantially yields

$$2(1-\alpha)\rho(\phi-1)+\hat{\sigma}_\eta^2\rho^2\frac{2\alpha^2\mu(l)^2-4\alpha\mu(l)+1}{(1-\alpha\mu(l))^2}+\lambda_1(l)-\lambda_2(l)+\lambda_3+c_l(\mu(l))=0.$$

Additional algebra shows that $(\phi-1)=-\frac{1}{\hat{k}+\hat{\sigma}_\eta^2-\alpha Q}\hat{\sigma}_\eta^2$. Using this result, the first order condition simplifies further to

$$-\hat{\sigma}_\eta^2\left(\frac{\rho}{1-\alpha\mu(l)}\right)^2+\lambda_1(l)-\lambda_2(l)+\lambda_3+c_l(\mu(l))=0. \tag{F.30}$$

Furthermore, algebraic manipulations of (F.25) establishes that in equilibrium $\lambda_3=0$

Let $\bar{\lambda}_1(l)=\lambda_1(l)\left(1-\alpha\mu(l)\right)^2$ and $\bar{\lambda}_2(l)=\lambda_2(l)\left(1-\alpha\mu(l)\right)^2$. Proposition 8 combines the above results to characterize the set of equilibria in the extended model.

Proposition 8. *The set of equilibria in the model are characterized by the set of equalities indexed by l,*

$$c_l(\mu(l))\left(1-\alpha\mu(l)\right)^2=\hat{\sigma}_\eta^2\rho^2-\bar{\lambda}_1(l)+\bar{\lambda}_2(l), \tag{F.31}$$

the inequality constraints

$$\mu(l)\le 1;\mu(l)\ge 0;\int_0^{\hat{n}}\mu(l)dl\le\hat{k}, \tag{F.32}$$

the complementarity slackness conditions, $\bar{\lambda}_1(l)(\mu(l)-1)=0$, $\bar{\lambda}_2(l)\mu(l)=0$, and the inequalities, $\bar{\lambda}_1(l)\ge 0,\bar{\lambda}_2(l)\ge 0$.

F.3 A Sufficient Condition for Uniqueness

Suppose that the cost of information is given by the CES aggregator in equation (5.2), with $\omega > 1$. The derivative of cost with respect to $\mu(l)$ is

$$c_l(\mu(l)) = \lambda \hat{n}^{\frac{\omega-1}{\omega}} \left(\int_0^{\hat{n}} \mu(l)^{\omega} dl \right)^{\frac{1-\omega}{\omega}} \mu(l)^{\omega-1}. \tag{F.33}$$

The model has a unique equilibrium whenever the left-hand side of (F.31) is monotonically increasing in $\mu(l)$. To see this, note first that one can immediately rule out $\mu(l) = 0$, since the derivative of the cost function with respect to $\mu(l)$ is always zero when $\mu(l) = 0$. Second, note that if $\mu(l) \in (0, 1)$ satisfies

$$c_l(\mu(l)) \left(1 - \alpha\mu_l\right)^2 = \hat{\sigma}_\eta^2 \rho^2 \tag{F.34}$$

for any k, then monotonicity implies that $c_l(\mu(l)) \left(1 - \alpha\mu_l\right)^2 > \hat{\sigma}_\eta^2 \rho^2$ at $\mu(l) = 1$, ruling out the possibility that $\lambda_1(l) \geq 0$, and therefore that $\mu(l) = 1$, for any k. Finally, when $\bar{\lambda}_1(l) = \bar{\lambda}_2(l) = 0$ and the lefthand side is monotonic, only one value $\mu(l)$ can simultaneously satisfy equation (F.31), so that $\mu(l) = \nu$ and the equilibrium conditions reduce to the those from the baseline model.

The required monotonicity is achieved whenever

$$\mu(l)^{\omega-1}(1 - \alpha\mu(l))^2 \tag{F.35}$$

is monotonic on $[0, 1]$. Taking a derivative and imposing the inequality quickly establishes the requirement that

$$\omega > \frac{1+\alpha}{1-\alpha}. \tag{F.36}$$

F.4 Multiple Equilibria when Information Cost is Linear

Suppose now that that the derivative of the cost function $c_l(\mu(l)) = \hat{\lambda}$. An immediate implication of proposition 8 is that, in equilibrium, the function $\mu(l)$ can take on no more than one interior value, in addition to $\mu(l) = 0$ or $\mu(l) = 1$. To see this, consider expression (F.31) for a value of l for which neither constraint one nor constraint two is binding. In this case, the left-hand side of 8 is strictly decreasing in $\mu(l)$, implying that no more than one interior value of $\mu(l)$ can simultaneously satisfy the equation. In contrast to case the case where the left hand side is increasing, however, it still may be that $\bar{\lambda}_1(l) > 0$ or $\bar{\lambda}_2(l) > 0$ or both, creating the potential for a great deal of multiplicity.

Imposing the restriction that $\mu(l)$ take on no more than one interior value, a set of simple conditions can be derived characterizing the set of equilibria in the model. Let $\hat{n}_1, \hat{n}_2, \hat{n}_3; \hat{n} \geq \hat{n}_i \geq 0; \hat{n} = \sum_{i=1}^{3} \hat{n}_i$ denote the "mass" of signals taking on values $\mu^* \in (0,1), \bar{\mu} = 1, \underline{\mu} = 0$, respectively. Solving the first order condition for μ^* yields

$$\mu^* = \frac{(1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2 - \hat{n}_2}{(1-\alpha)\hat{n}_1 - \alpha(\hat{n}_2 + \hat{\sigma}_\eta^2)}. \tag{F.37}$$

Since $\{\hat{n}_1, \hat{n}_2\}$ imply values for μ^* and $\hat{n}_3$, they are sufficient to characterize all equilibria. Assumption 1 ensures that agents acquire at least some information, so that $\hat{n}_1 + \hat{n}_2 > 0$. Furthermore, if $\hat{n}_1 + \hat{n}_2 < \hat{n}$, the requirement that $\lambda_1(l) \geq 0$ and $\lambda_2(l) \geq 0$ ensures $\mu^* \in [0,1]$. Proposition 9 describes the necessary and sufficient conditions for this.

Proposition 9. *Suppose that the cost of information is given by $c(\mu(l)) = \lambda\hat{k}$. The the set of equilibria is characterized by $\{\hat{n}_1, \hat{n}_2\}$ that satisfy one of the two sets of conditions below*

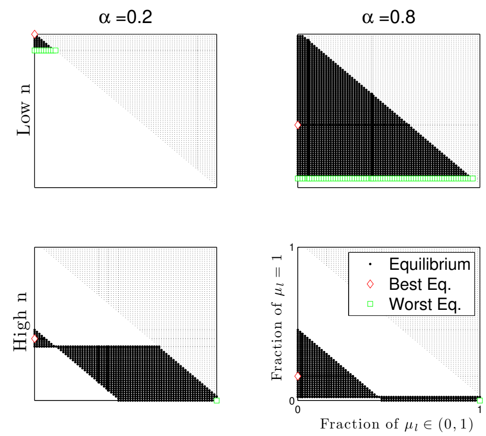

Figure 7: Multiple equilibria for different degrees of scope and strategic complementarity. The characteristics of the "best" and "worst" equilibria from the social planners perspective depend on the degree of scope.

- *Case 1: Full Acquisition Only:* $\hat{n} \leq (1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2$

  1. $\hat{n}_1 = 0$ *and* $\hat{n}_2 = \hat{n}$

- *Case 2: Multiple Equilibria:* $\hat{n} > (1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} - \hat{\sigma}_\eta^2$

  – *Case 2a*

    1. $(1-\alpha)\hat{n}_1 < \alpha(\hat{n}_2 + \hat{\sigma}_\eta^2)$
    2. $\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \geq \hat{\sigma}_\eta^2 + \hat{n}_1 + \hat{n}_2$
    3. $(1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \leq \hat{\sigma}_\eta^2 + \hat{n}_2$

  – *Case 2b*

    1. $(1-\alpha)\hat{n}_1 > \alpha(\hat{n}_2 + \hat{\sigma}_\eta^2)$
    2. $\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \leq \hat{\sigma}_\eta^2 + \hat{n}_1 + \hat{n}_2$
    3. $(1-\alpha)\left(\frac{\hat{\sigma}_\eta^2}{\hat{\lambda}}\right)^{\frac{1}{2}} \geq \hat{\sigma}_\eta^2 + \hat{n}_2$

The two-by-two panel in figure 7 captures the range of multiple equilibria for different degrees of strategic complementarity, at different levels of scope. In the first row, with relatively low scope, equilibrium always entails some degree of perfectly directed search by agents. This result is closely related to the result in baseline model where, for low levels of scope, agents always choose to observe all signals. In this case, if agent i can be assured that no others observe a particular signal k, then she may choose not to observe it as well. However, if others do observe that signal with positive probability, then she desires to do so as well, which increases the signal's informativeness, causing others to increase the probability with which they draw that signal, and so on. Once agents have acquired enough signals in this directed manner, however, this logic no longer bites, and agents may choose undirected search over some of the remaining signals.

Higher strategic complementarities generally increase the scope for multiplicity. In the current model, this effect is apparent when scope is low. When scope is high, however, the consequences for multiplicity are more subtle. This contrast stems from the fact that, when complementarities are weak, agents are roughly indifferent to the degree of coordination in their information. As a result, agent i is much less responsive in her own information choice to the degree to which other agents are direct their search. As a result, the set of equilibria under weak complementarity includes a range of information-search profiles that would be eliminated if agents had stronger strategic incentives.

References

Chen, X.-H., A. P. Dempster, and J. S. Liu (1994). Weighted Finite Population Sampling to Maximize Entropy. *Biometrika 81*(3), pp. 457–469.

Judd, K. L. (1985). The Law of Large Numbers With a Continuum of IID Random Variables. *Journal of Economic Theory 35*(1), 19–25.

McCall, J. J. (1991). Exchangeability and Its Economic Applications. *Journal of Economic Dynamics and Control 15*(3), 549 – 568.

Myatt, D. P. and C. Wallace (2012). Endogenous Information Acquisition in Coordination Games. *The Review of Economic Studies 79*(1), 340–374.

Uhlig, H. (1996). A Law of Large Numbers For Large Economies. *Economic Theory 8*(1), 41 – 50.